EXPLORING

PHYSICS

YEAR 12 - EXPERIMENTS, INVESTIGATIONS & PROBLEMS

Worked Solutions

The STAWA *Worked Solutions* have been developed through the collaboration of teachers working in Department of Education, Catholic Education WA and Association of Independent Schools of WA. Funding assistance was provided by the Department of Education.

The *Worked Solutions* are intended to support the problem sets of the STAWA ATAR Exploring Physics Year 12: experiments, investigations and problems.

In an endeavour to provide the highest quality publication, the STAWA *Worked Solutions* were written and checked by different teachers. This does not guarantee that all answers are correct. Teachers are advised to work through disputed solutions with their students. If they are sure there is an error then they are asked to forward corrections to STAWA by email: admin@stawa.net

The STAWA *Worked Solutions* are a great example of teachers helping teachers for the benefit of all students.

STAWA
SCIENCE TEACHERS' ASSOCIATION OF WESTERN AUSTRALIA

YEAR 12 ATAR

Gravity and Motion

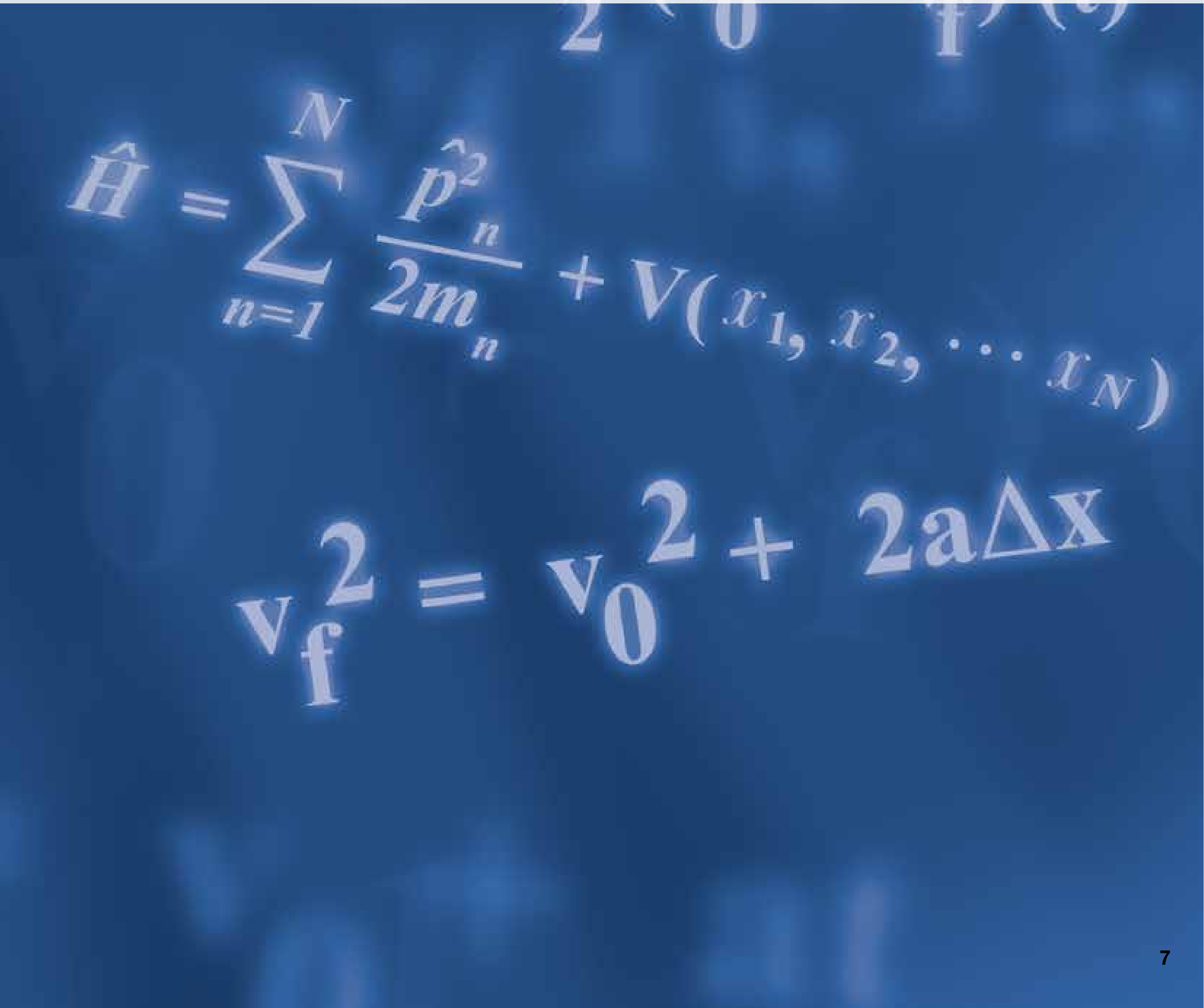
Develop a deeper understanding of motion, its causes and the field theory of gravity through investigations.

Use Newton's Laws of Motion and the gravitational field model to analyse:

- **motion on inclined planes,**
- **the motion of projectiles, and**
- **satellite motion.**

Investigate to develop skills in relating graphical representations of data to quantitative relationships between variables, using lines of force to represent vector fields, and interpreting interactions in two and three dimensions.

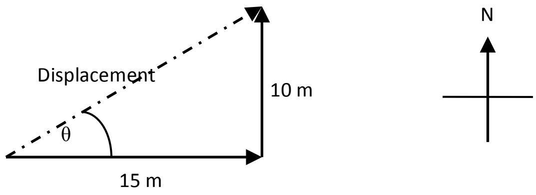
Explore the ways in which models and theories are related to gravity and motion by investigating contexts including technologies, such as artificial satellites, navigation devices, and related areas of science and engineering, such as sports science, amusement parks, ballistics and forensics.



Problem Set 1: Vector additions, subtractions and resolution

1. Velocity, speed in a given direction, is a vector quantity and as such has both magnitude and direction.

2. a) Players displacement?

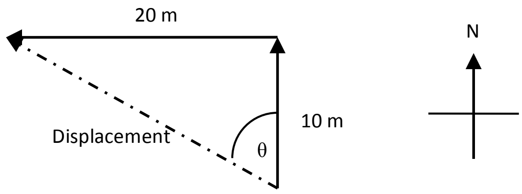


$s = \sqrt{(15^2 + 10^2)}$
$s = 18.0$ m

$\theta = \text{Arctan}\ (10 / 15)$
$\theta = 33.7^0$

$s = 18.0$ m East 33.7^0 North **or** **$s = 18.0$ m North 56.3^0 East.**

b) Ball displacement



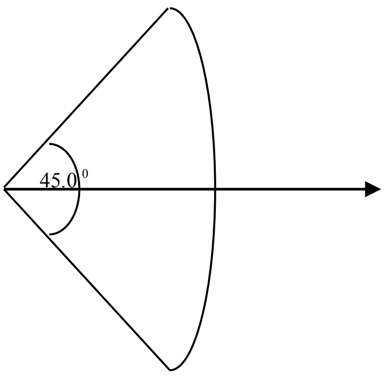
$s = \sqrt{(10^2 + 20^2)}$
$s = 22.4$ m

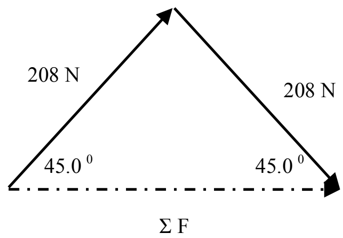
$\theta = \text{Arctan}\ (20 / 10)$
$\theta = 63.4^0$

$s = 22.4$ m North 63.4^0 West **or** **$s = 22.4$ m West 26.6^0 North**

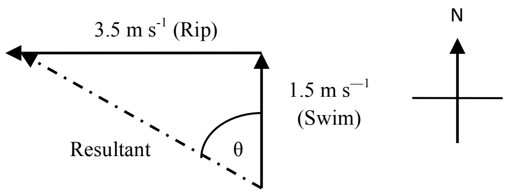
3. Force on arrow

$\Sigma F = \sqrt{(208^2 + 208^2)}$
$\Sigma F = 294$ N Forwards





4. Assuming the swimmer is swimming north …



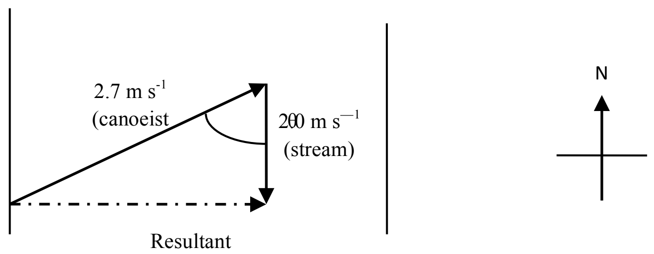
$v = \sqrt{(3.5^2 + 1.5^2)}$
$v = 3.81$ m s^{-1}

$\theta = \text{Arctan}\ (3.5 / 1.5)$
$\theta = 66.8^0$

$v = 3.81$ m s^{-1} North 66.8^0 West **or** **$v = 3.81$ m s^{-1} West 23.2^0 North.**

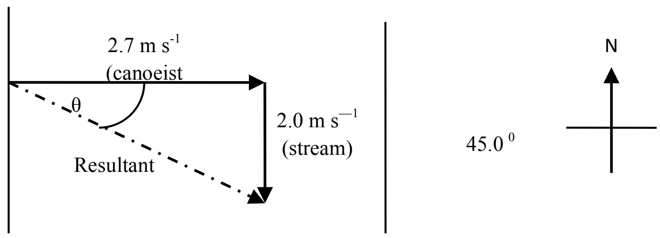
5. Canoeist

a) θ = Arcos (2.0 / 2.7)
θ = 42.2^0 to the current
(θ = 47.8^0 to the bank)



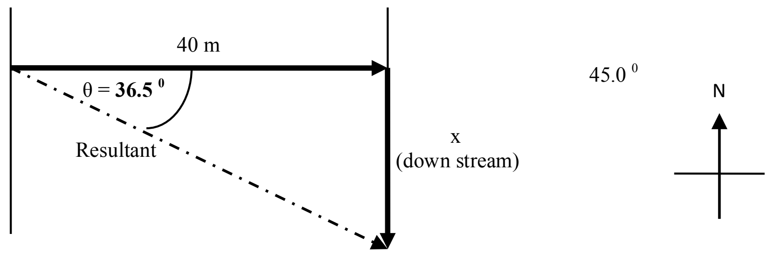
b) $v = \sqrt{(2.7^2 + 2.0^2)}$
v = 3.36 m s^{-1}

θ = Arctan (2.0 / 2.7)
θ = 36.5^0

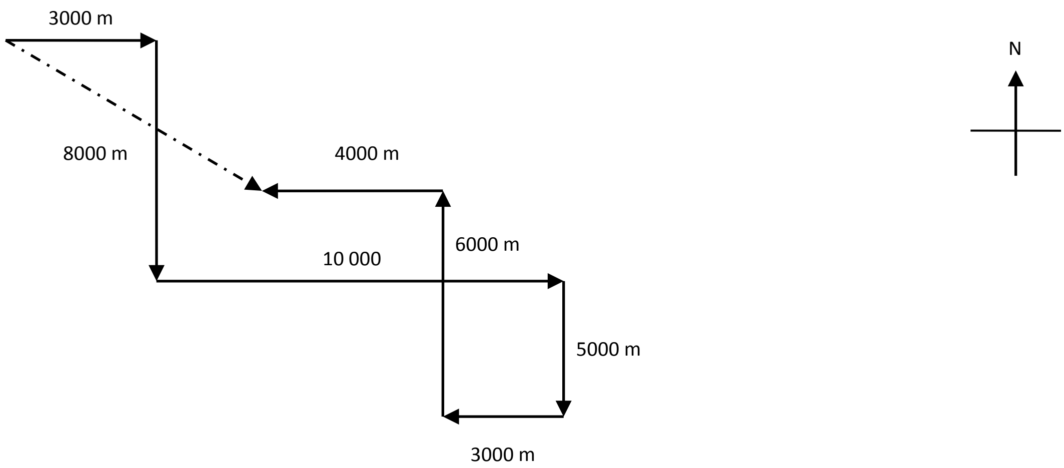


v = 3.36 m s^{-1} at 53.5^0 to the bank and angled downstream.

c)
Tan 36.5 = (x / 40)
x = 29.6 m down stream



6. a) Cross country skier: This diagram is **not** to scale.
An appropriate scale for a student to use would be 1 cm = 1 000 m



Add north – south and east - west vectors separately using a sign convention.

	North – South (+ / -)	East – West (+ / -)
	- 8000 m	+ 3000
	- 5000 m	+ 10 000
	+ 6000 m	- 3000
		-4000
Total:	**-7 000 m**	**+ 6 000 m**

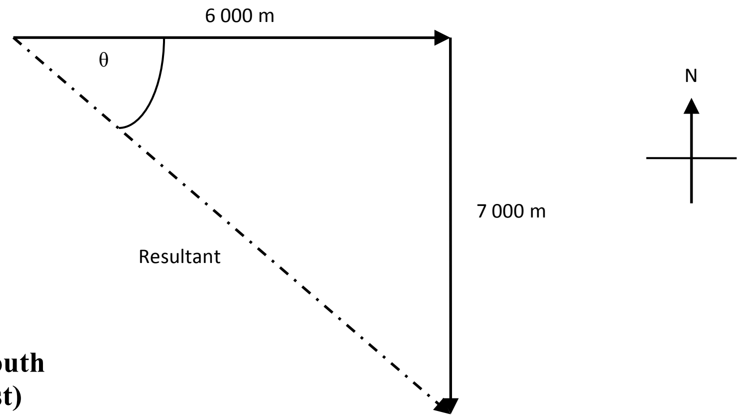
6. a) continued

$s = \sqrt{(6\,000^2 + 7\,000^2)}$

$s = 9.22 \times 10^3$ m

$\theta = \text{Arctan}\ (7\,000 / 6\,000)$

$\theta = 49.4^0$



$s = 9.22 \times 10^3$ m East 49.4^0 South
(9.22×10^3 m South 40.6^0 East)

b) Return journey is $s = 9.22 \times 10^3$ m North 40.6^0 W

7. Let towards the netballer be the positive direction and away negative.

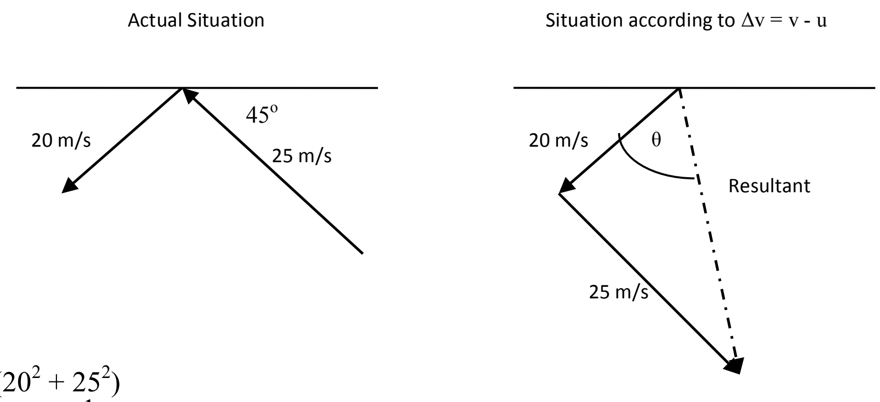
$\Delta v = v - u$
$\Delta v = [0 - (+5)]$
$\Delta v = - 5$ m s^{-1} or 5 m s^{-1} backwards

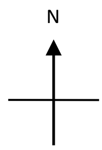
8. Let towards the tennis player be positive and away from the player be negative.

$u = + 80 / 3.6 = +22.2$ m s^{-1}
$v = - 90 / 3.6 = -25.0$ m s^{-1}

$\Delta v = v - u$
$\Delta v = [(-25) - (+22.2)]$
$\Delta v = [(-25) - 22.2]$
$\Delta v = - 47.2$ m s^{-1} or 47.2 m s^{-1} away from the player.
Or -170 km h^{-1}

9.



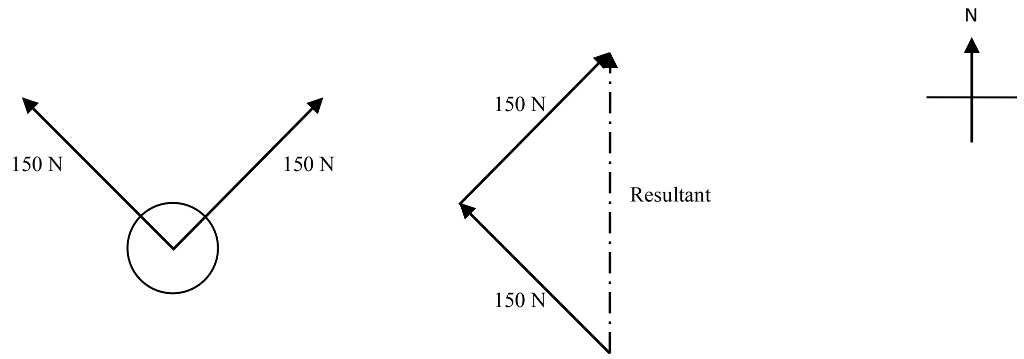


$\Delta v = \sqrt{(20^2 + 25^2)}$
$\Delta v = 32.0$ m s^{-1}

$\theta = \text{Arctan}\ (25 / 20)$
$\theta = 51.3^0$

$\Delta v = 32.0$ m s^{-1} at $\theta = 51.3^0$ to the final velocity vector or at 96.3^0 to the wall.

10.



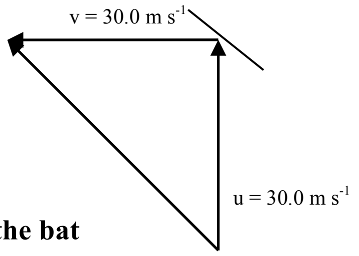
$F = \sqrt{(150^2 + 150^2)}$
$F = 212$ N

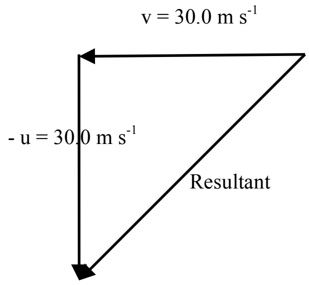
11.

$\Delta v = \sqrt{(30.0^2 + 30.0^2)}$
$\Delta v = 42.4$ m s^{-1}

θ = at right angles to the bat

v = 42.4 m s^{-1} at right angles to the bat





12.
Position 1
Towards the moon is positive. Towards the earth is negative.

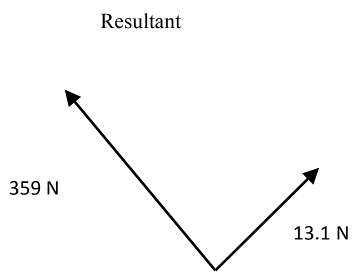
$\Sigma F = (-480) + (+53.2)$
$\Sigma F = -426.8$ N
$\Sigma F = 426.8$ N towards Earth

Position 2

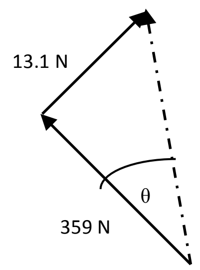
$\Sigma F = \sqrt{(359^2 + 13.1^2)}$
$\Sigma F =$ **359.2 N**

$\tan \theta = 13.1 / 359.$
$\theta = 2.09^0$

Actual Situation



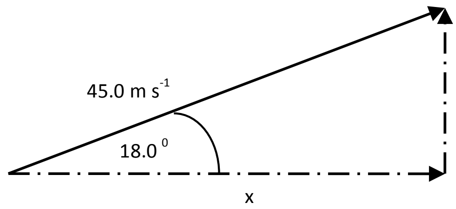
Situation according to Σ F



$\Sigma F =$ **359 N at 2.09^0 away from the line joining the asteroid to the earth. Angle bends towards the moon.**

13.

$\cos 18^0 = x / 45$
$x = 45 \cos 18^0$
$v = 42.8$ m s^{-1} horizontally

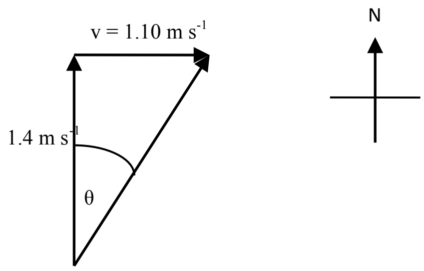


14.

$v = \sqrt{(1.4^2 + 1.1^2)}$
$v = 1.78$ m s^{-1}

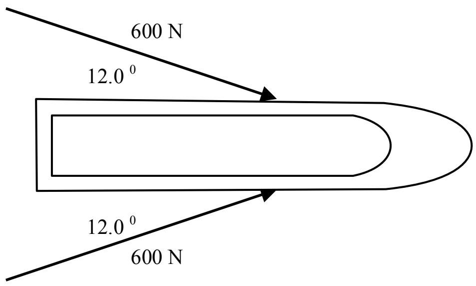
$\tan \theta = 1.1 / 1.4$
$\theta = 38.2^0$

$v = 1.78$ m s^{-1} North 38.2^0 East



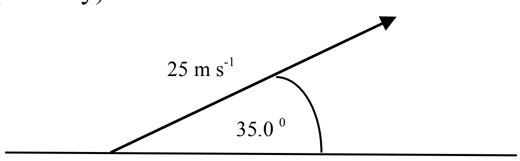
15. a)

$F = 2 \times (600 \cos 12)$
$F = 1173.8$ N
$F = 1.17 \times 10^3$ N Forward



b) $600 \sin 12 = 125$ N

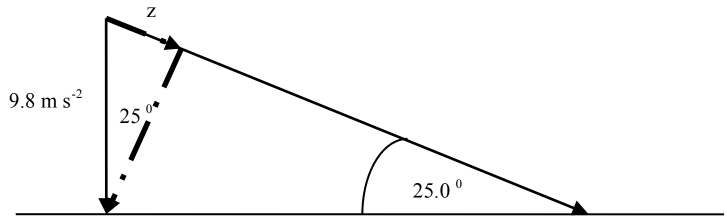
16. For rate is vague rate of change of displacement (velocity)



(ignoring air resistance)	Vertical	Horizontal towards the pin
Velocity	Initially = 25 sin (35) = 14.3 m/s and then decreasing due to gravitational acceleration.	25 cos (35) = 20.5 m/s and holding constant.

17.

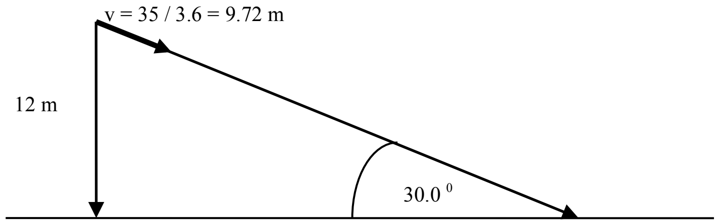
sin 25 = z / 9.8
z = 4.14 m s^{-2}
a down the slope = 4.14 m s^{-2}



18.

Distance down the slope = ?
s = 12 / sin 30
s = 24 m down the slope

v = s / t
t = s / v
t = 24 / 9.72
t = 2.47 s

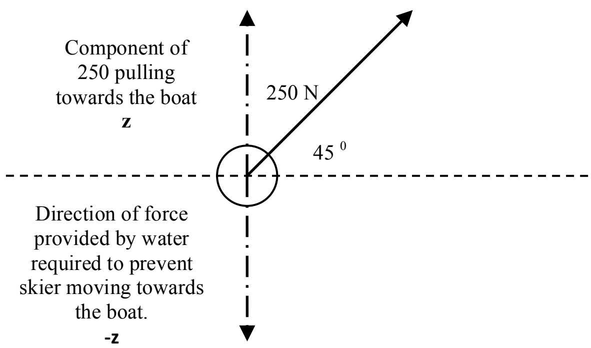


19.

Force pulling towards a boat
z = 25 sin 45
z = 17.7 N towards the boat

-z = 17.7 N = 17.7 N away from the boat

Top View



20. a)

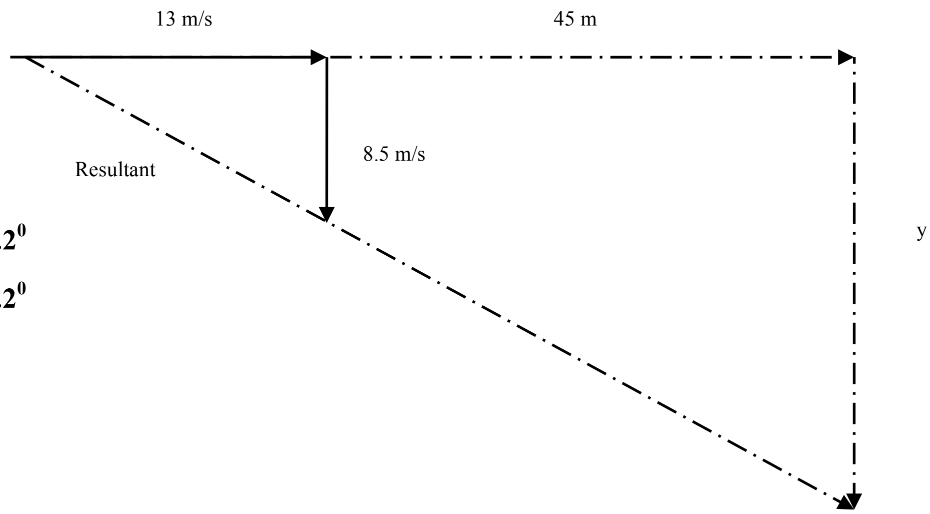
$v = \sqrt{(13^2 + 8.5^2)}$
v = 15.5 m s^{-1}

Tan θ = 8.5 / 13
θ = 33.2^0

$s_{hypotenuse}$ = 45 / cos 33.2^0

$s_{hypotenuse}$ = 45 / cos 33.2^0
$s_{hypotenuse}$ = 53.77 m

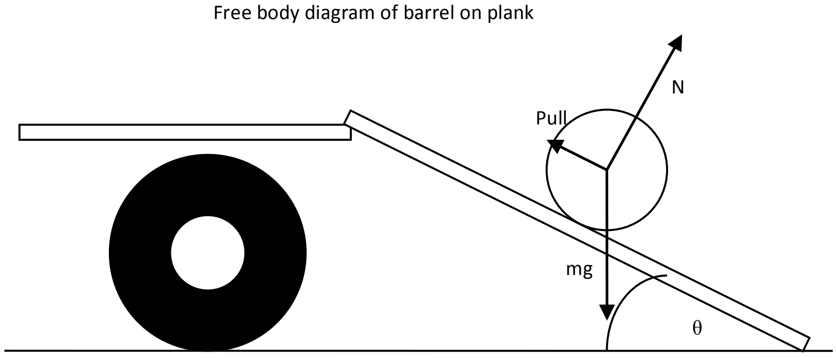
v = s / t
t = s / v
t = 53.77 / 15.5
t = 3.47 s

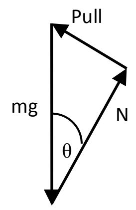


b)
tan (33.2) = y / 45
y = 45 tan 33.2
y = 29.5 m

21. a) The plank could be set at an angle to reduce the normal force that is required.

b) When the normal force on the plank = 166 kg x 9.8 = 1626.8 N the plank breaks



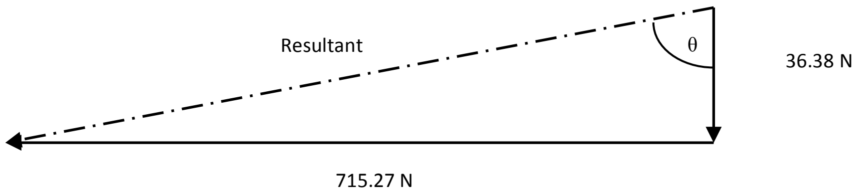


Drum Weight = 197 x 9.8 = 1930.6 N

Cos θ = N / mg
θ = Arc cos (1626.8 / 1930.6)
θ = 32.6^0

22. Skiers behind boat. Force on boat = ?

	Y axis (+/-)	**X axis (+/-)**
220 at 30^0 above	220 sin 30 = 110 N	-220 cos 30 = -190.52 N
180 at 12^0 above	180 sin 12 = 37.42 N	-180 cos12 = -176.07 N
170 at 10^0 below	-170 sin 10 = -29.52 N	-170 cos10 = -167.42 N
200 at 25^0 below	-200 sin 25 = - 84.52	-200 cos25 = -181.26 N
Total	+36.38 N	-715.27 N

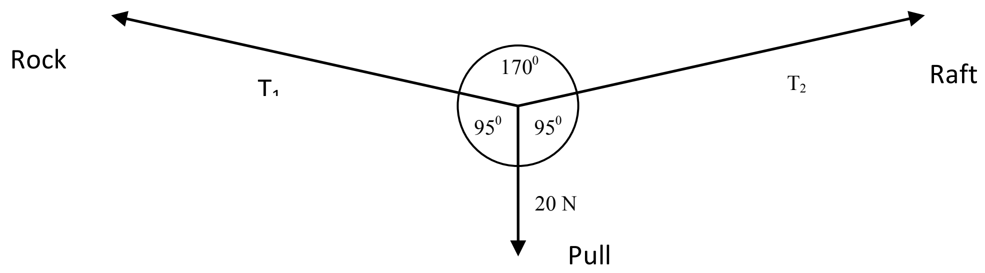


$\Sigma F = \sqrt{(36.38^2 + 715.27^2)}$
ΣF = 716 N

Tan θ = 715.27 / 36.38
θ = 87.1^0

ΣF = 716 N at 87.1^0 to the back of the boat or 2.91^0 below the mid line.

23.



Using sin rule
$20 / \sin 170^o = T_2 / \sin 95^o$

T_2 = 115 N **The closer the 170^0 angle is to 180^0, the larger the Tension in the wire.**

Problem Set 2: Moments and equilibrium

1. m N is N m which is newton meters. These are the units for torque (moment)
   mN is milli newtons. This is newtons x 10^{-3}.

2. M = F x r
   M = 160 x 0.75
   M = 120 N m
   (direction cannot be specified other than having a knowledge of nuts and bolts. To tighten bolts you must turn them clockwise).

3. M = F x r
   88= F x 0.4
   F = 220 N at right angles to the wrench.

4.

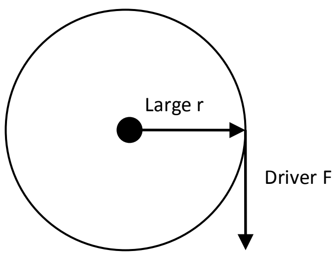
Truck

The truck has a very "heavy" steering mechanism. As a consequence it will require a lot of torque to get it to turn.

By increasing the diameter of the steering wheel, the torque that can be created when the driver applies their force to the wheel will be maximised.

The speed with which the driver is required to turn the wheel is not a major consideration

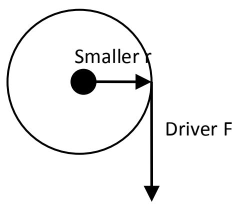
A large wheel provides a force (torque advantage).



Car

The racing car has a very light steering mechanism. What is important is that the driver be able to respond quickly to changes in conditions in front of them. A small steering wheel does not have to be turned through a large distance in order to bring about a change in the direction of the vehicle.

A small radius steering wheel provides a shortness of turning distance advantage.

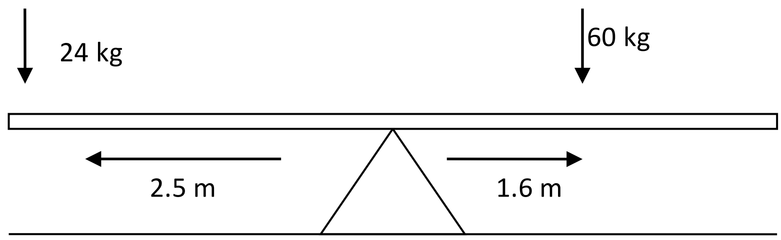


5. Michael leans forward to keep the combined centre of mass (his com and the backpacks com combined) above his base. This causes the weight vector of the combined com to act through the base eliminating any toppling torque. If he does not lean forward the weight of the combined centre of mass acts outside his base (feet behind the heels) and this causes him to topple over backwards.

Lean Forwards | Stands straight

Does not topple – weight within base | Does topple – weight outside of base

6.a)

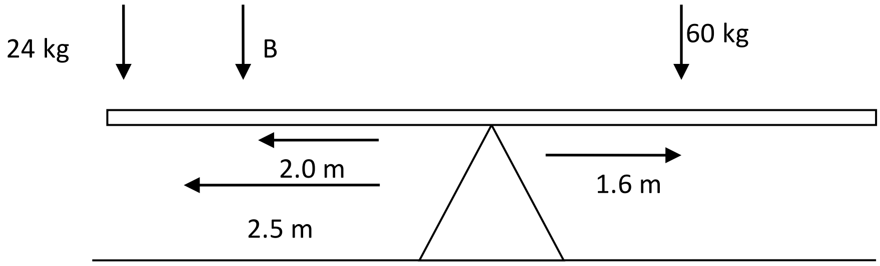


$\Sigma M_c = 60 \times 9.8 \times 1.6$
$\Sigma M_c = 940.8$ N m Clockwise

$\Sigma M_a = 24 \times 9.8 \times 2.5$
$\Sigma M_a = 588.0$ N m Anti clockwise

940.8 N m Clockwise $\neq$ 588.0 N m Anti clockwise and so cannot reach a balance.

6.b)



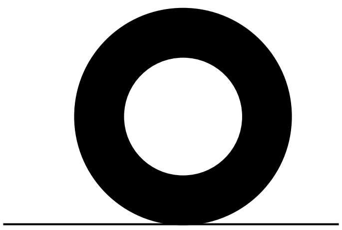
$\Sigma M_c = \Sigma M_a$

(60 x 9.8 x 1.6) = (24 x 9.8 x 2.5) + (B x 9.8 x 2.0)
940.8 = 588.0 + 19.6 B
940.8 - 588.0 = + 19.6 B
940.8 - 588.0 = + 19.6 B
352.8 / 19.6 = B

B = 18.0 kg

7. In both situations the torque provided by the motor is the same.

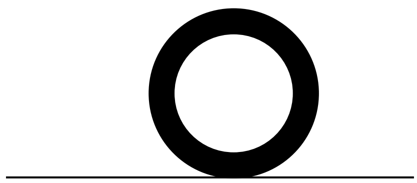
Standard Tyres (large radius)



M (constant) = $r\uparrow F\downarrow$.

For a constant torque the larger the radius the smaller the force. The smaller the force the smaller the acceleration of the car by F = ma

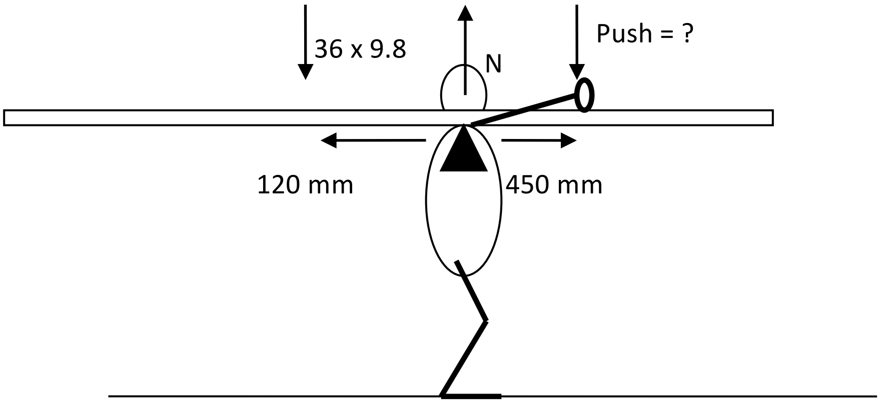
Low Profile Tyres (smaller radius)



M (constant) = $r\downarrow F\uparrow$.

For a constant torque the smaller the radius the larger the force. The larger the force the greater the acceleration of the car by F = ma

8.



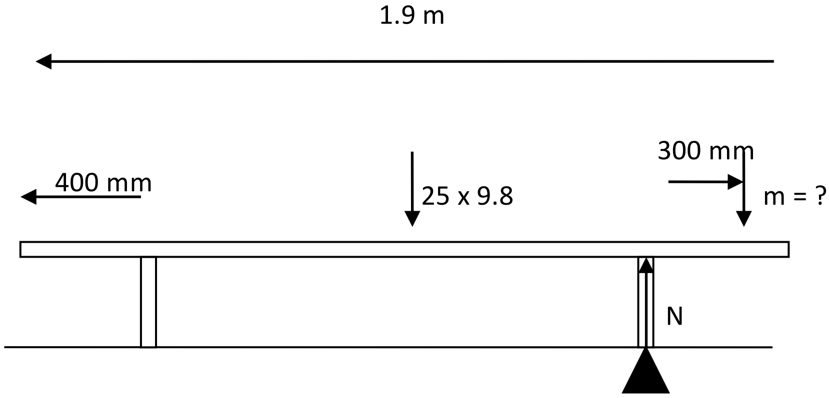
a)
Σ M c = Σ M a
Σ 0.120 x 36 x 9.8 = Push x 0.450

Push = 94.1 N down

b)
Σ F up = Σ F down
N = 36 x 9.8 + 94.1 N

N = 447 N up

9.

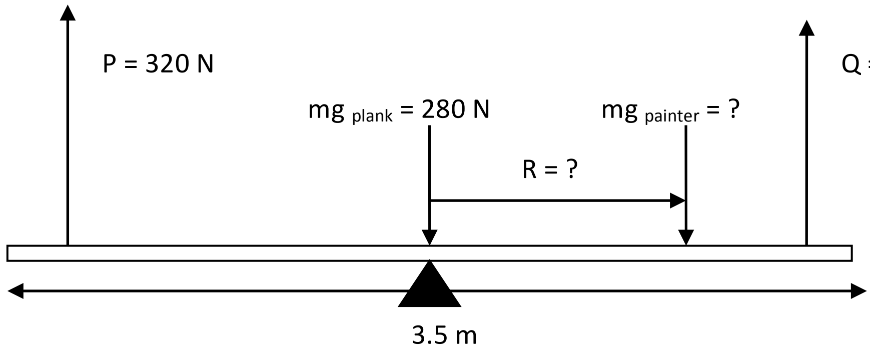


The pivot must be placed at the leg because the seat is in unstable equilibrium

Σ M a = Σ M c
0.55 x 25 x 9.8 = m x 9.8x 0.300

m = 46 kg

10.



a)

Σ F up = Σ F down   320 + 590 = 280 + mg $_{painter}$ **mg $_{painter}$ = 630 N Down**

b) Take moments about the centre of the platform. Assume that the ropes are attached to the ends of the plank.

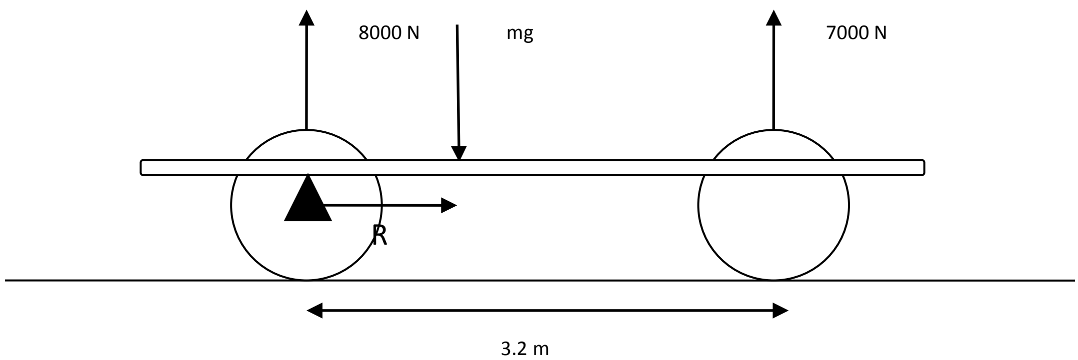
Σ M c = Σ M a   [320 x (3.5/2)] + [630 x R] = [590 x (3.5/2)]

560 + 630R = 1032.5

$$R = \frac{1032.5 - 560}{630}$$

R = 0.75 m towards Q

11.



Σ F up = Σ F down

8000 + 7000 = mg

mg = 15000 N Down

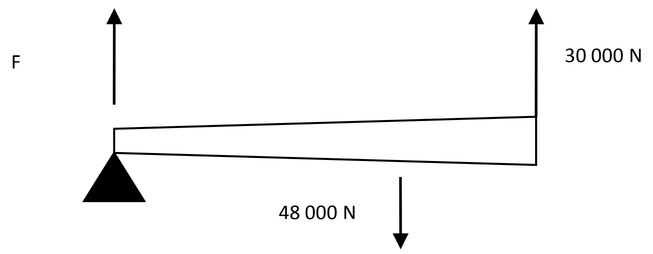
Take moments about front wheel of car.

Σ M c = Σ M a

[15000 x (R)] = [7000 x 3.2]

R = 1.49 m from the front wheel

12. a) Σ F up = Σ F down
30 000 + F = 48 000
F = 18 000 N

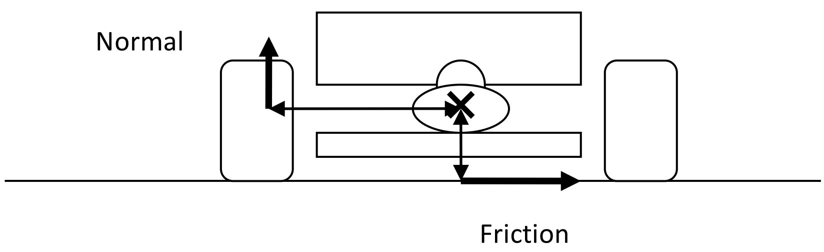


b) Let the length of the log be 1.00 m. Take moments about the light end
Σ M c = Σ M a [48 000 x (R)] = [30 000 x 1]
R = 0.625 m from the light end (R = 0.375 m from the heavy end)
(62.5 % from the light end or 37.5 % from the heavy end if length is uspecified.)

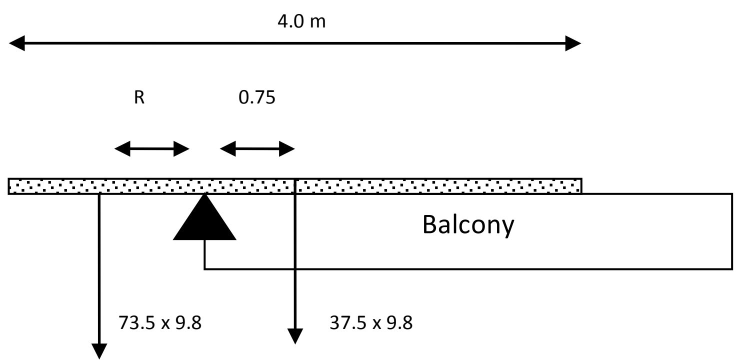
13. When a car goes around a corner on a flat road it is the outside tyres that tend to provide the centripetal force by friction required to round the bend. If the torque provided by the outer tyres accelerating towards the centre of the curve is greater than the torque provided by the normal force acting on the tyre, the racing car will roll over.

Because the racing car is accelerating around the bend, torques must be taken about the centre of mass of the object. This is different to when the object is

- in stable equilibrium – the pivot can be chosen arbitrarily.
- in unstable equilibrium – the pivot is taken about the point base.



14. a)
Torques must be taken about the edge of the balcony because just as the plank is about to tip the plank is in unstable equilibrium.



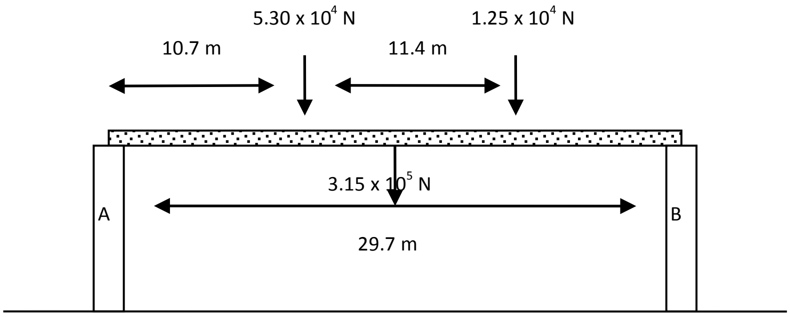
Σ M c = Σ M a

[37.5 x 9.8 x 0.75] = [73.5 x 9.8 x R]
[275.625] = [720.3 x R]
$\frac{275.625}{720.3}$ = x R

R = 0.383 m from the edge of the balcony

b) Move the com of the plank further form the edge or put the paint can at the opposite end of the plank to add extra stabilising torque to the plank.

15.



Take moments about pier A.
Σ M c = Σ M a
$(5.3 \times 10^4 \times 10.7) + (1.25 \times 10^4 \times (10.7 + 11.4)) + (3.15 \times 10^5 \times (29.7/2)) = [B \times 29.7]$
$(5.671 \times 10^5 + 2.7625 \times 10^5 + 4.67775 \times 10^6) = 29.7$ B
B = 1.86×10^5 N up

Σ F up = Σ F down
A + **1.86×10^5** = $5.3 \times 10^4 + 1.25 \times 10^4 + 3.15 \times 10^5$
A = 1.95×10^5 N up

16. With the backs of their legs to the wall, as they bend forward, the centre of mass of their body is put outside their base and they topple forward. They can only achieve this if they can keep the com inside their base (feet) at all times which is impossible … so they fall / topple.

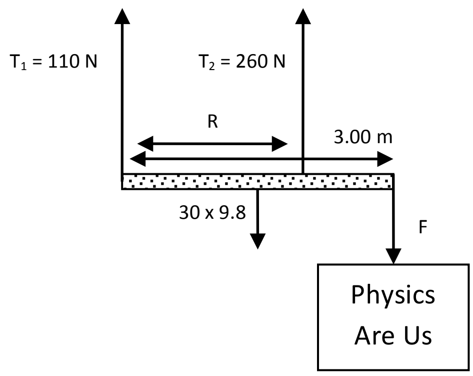
You should draw a diagram to support your answer.

17. While your com is closer to the ground, the size of you base (hands) is much smaller than usual (your feet). This results in you (the handstand) being less stable than usual. You could do a toppling angle analysis on the basis of (base / height to com) = tan θ

18. You need to shift your centre of mass from side to side to keep it above the foot that is on the ground to avoid inducing a torque that causes you to topple.

19. By leaning forward as you pass over the hurdle, it minimises the fluctuation in the change in height of the centre of mass of the hurdler. When the com of the hurdler rises, the potential energy of the hurdler increases. By the law of conservation of energy, if your potential energy increases then your kinetic energy and consequently velocity decreases. A slow velocity causes you to travel the distance of the race in a longer time. Hence you have a greater chance of losing.

20.

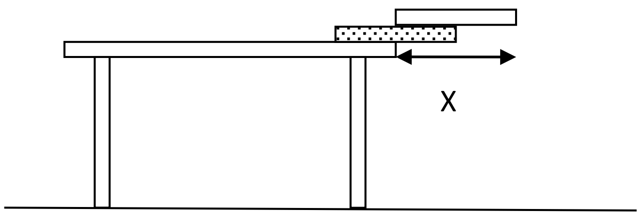


Σ F up = Σ F down
110 + 260 = 294 + F
F = 76 N Down

m =F / g
m = 76 / 9.8
m = 7.76 kg

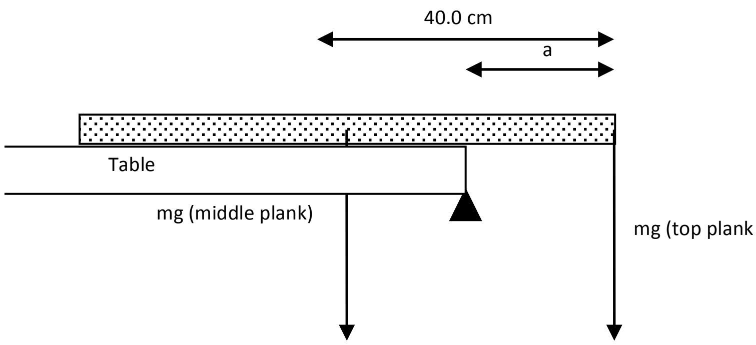
Σ M c = Σ M a
(294 x 1.5) + (76 x 3) = [260 x R]
441 + 228 = 260 R
R = 669 / 260
R = 2.57 m from T_1 end

21. Hint - analyse the plank that is in between the table and the top plank because it touches all other objects.



Place the COM of the top plank at the edge of the middle plank.
The pivot is at the edge of the table because the system is in unstable equilibrium.

The free body diagram of the middle plank (close up view) is thus …



Σ M c = Σ M a
[mg x a] = [mg x (0.4 – a)]
mga = 0.4mg - mga
2mga = 0.4mg
2a = 0.4
a = 0.4 / 2
a = 0.2 m

The plank on top of the middle plank sticks out an additional 0.4 m.

Hence the distance X is 0.6 m (0.4 + 0.2)

22. The critical factor is that the centre of mass of the wood bottle system is above the base so that the weight force acts through the base. If the weight force did not act through the base, a torque would be induced that would topple the system.

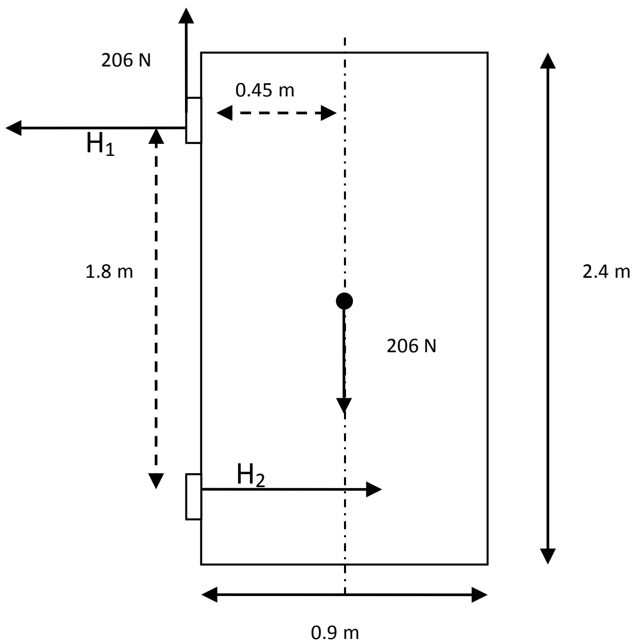
23. a)

Take moments about top hinge

$\Sigma M_c = \Sigma M_a$

$0.45 \times 206 = 1.8 \times H_2$

H_2 = 51.5 N to the right (diagram)



23. b)

$\Sigma F_{left} = \Sigma F_{right}$

$H_1 = H_2$

H_1 = 51.5 N Left

$\Sigma F_{up} = \Sigma F_{Down}$

103 + 103 = 206

H_1 Resultant

Magnitude

H1 Resultant^2 = $(103^2 + 51.5^2)$

H_1 Resultant = 115 N

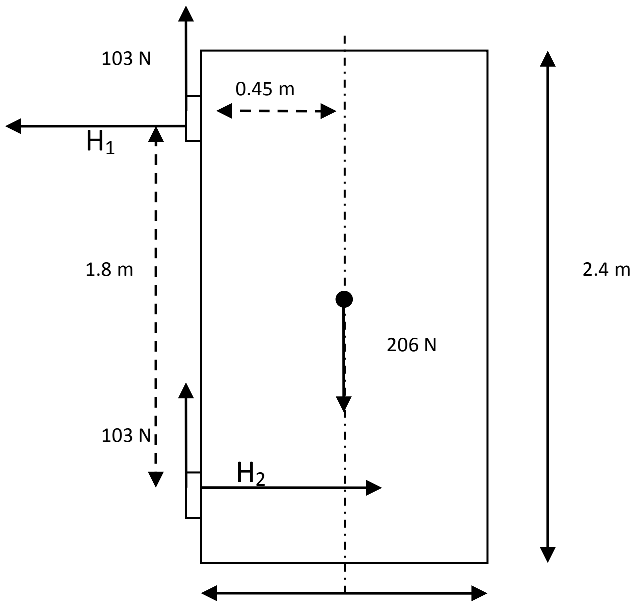
Angle

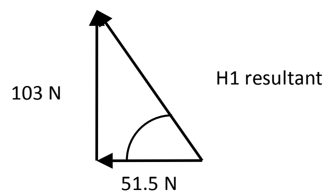
Tan θ = 103 / 51.5

θ = 63.4 ^{0}

Ans H_1 Resultant = 115 N left 63.4 ^{0} up

H_2 by symmetry is 115 N right 63.4 ^{0} up.





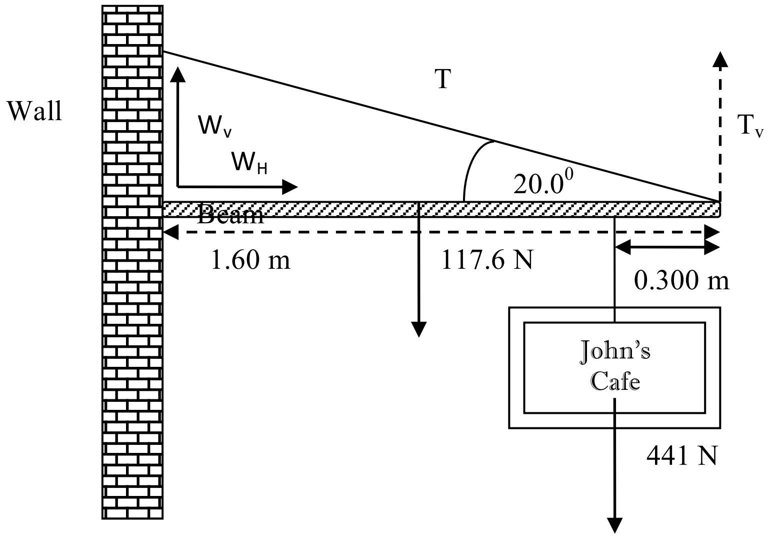
24. **a)** Take moments about hinge at wall beam intersection

$\Sigma M_c = \Sigma M_a$

$(0.8 \times 117.6) + (1.3 \times 441) = (1.6 \times T_v)$

$(94 + 573.3) / 1.6 = T_v$

$\mathbf{T_v = 417\ N}$



$\cos 70 = T_v / T$

$T = 417 / \cos 70$

$\mathbf{T = 1219\ N}$

$\mathbf{T = 1.22 \times 10^3\ N}$

along the wire.

b) $\Sigma F_{up} = \Sigma F_{down}$

$W_v + 417 = 117.6 + 441$

$\mathbf{W_v = 141.6\ N}$

$\Sigma F_{left} = \Sigma F_{right}$

$T_h = W_h$

$T \cos 20 = W_h$

$1219 \cos 20 = W_h$

$\mathbf{W_h = 1145\ N}$

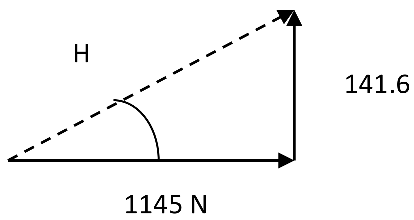
c)

$W = (1145^2 + 141.6^2)$

$W = 1153.7\ N$

$\tan \sigma = 141.6 / 1145$

$\sigma = 7.05^0$



W = 1.15 x 10^3 N Right 7.05^0 Up

25. **Find Tension**

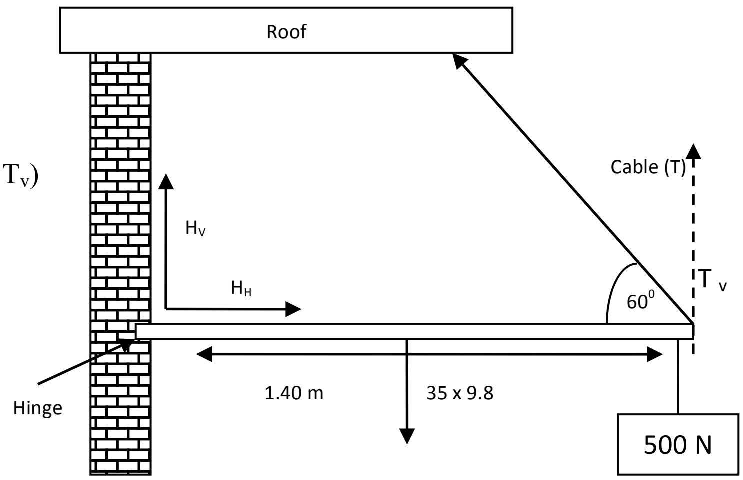
Take moments about hinge

$\Sigma Mc = \Sigma Ma$

$(0.7 \times 35 \times 9.8) + (500 \times 1.4) = (1.4 \times T_v)$

$(240.1 + 700) / 1.4 = T_v$

$T_v = \mathbf{671.5\ N}$



$\cos 30 = T_v / T$

$\cos 30 = 671.5 / T$

$T = 671.5 / \cos 30$

T = 775.4 N along the wire

Find the force of the hinge on the beam

$\Sigma F\ up = \Sigma F\ down$

$H_v + T_v = (35 \times 9.8) + (500)$

$H_v + 671.5 = (343) + (500)$

H_v = 171.5 N up

Σ F left = Σ F right

$H_h = T_h$

Cos 60 = T_h / T

T_h = T x Cos 60

T_h = 775.4 Cos 60

T_h = 387.7 N Left

$H_h = T_h$

H_h = 387.7 N Right

Combine the H_v and H_h

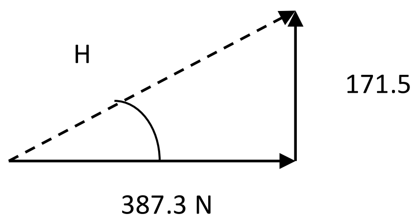
Pythagoras

$H = (171.5^2 + 387.7^2)^{½}$

H = 424 N

Tan θ = 171.5 / 387.3

$θ = 23.9^0$



Answer = W = 424 N Right 23.9^0 Up

26. Σ Mc = Σ Ma

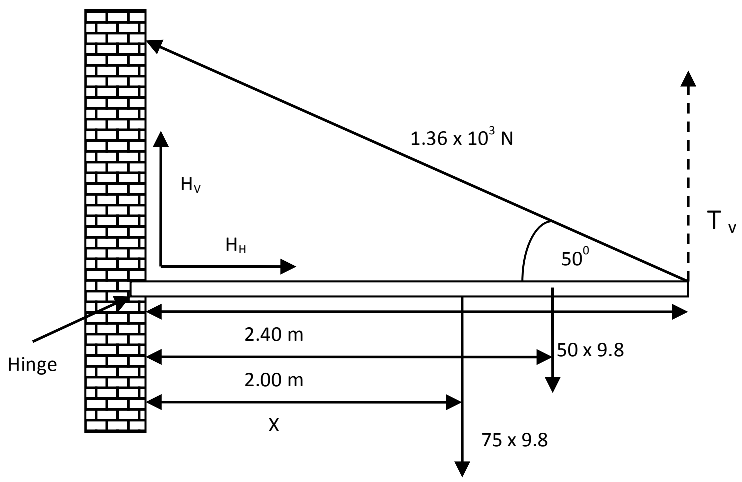
$(2.00 \times 50 \times 9.8) + (X \times 75 \times 9.8) = (2.4 \times 1.36 \times 10^3 \times \cos 40^0)$

980 + 735X = 2500

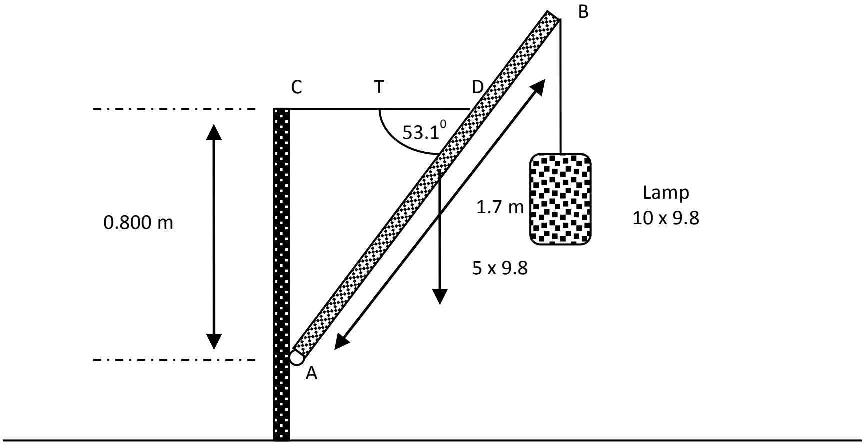
735X = 2500 - 980

X = (2500 – 980) / 735

X = 2.07 m



27.



a) Find tension

Σ Mc = Σ Ma

([1.7 / 2] Cos53.1^0 x 5 x 9.8) + (1.7 Cos 53.1^0 x 10 x 9.8) = (T x 0.8)

25 + 100 = 0.8 T

125 / 0.8 = T

T = 156 N

b) Forces at A

Σ F up = Σ F down

A_v = (5 x 9.8) + (10 x 9.8)

A_v = 147 N up

Σ F left = Σ F right

T = A_h

A_h = 156 N Right

Combine the A_h and A_v

Pythagoras

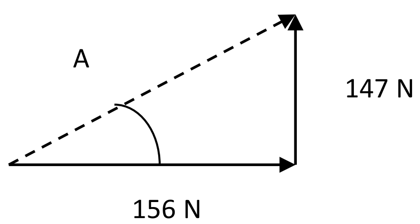
A = $(156^2 + 147^2)^{½}$

A = 214 N

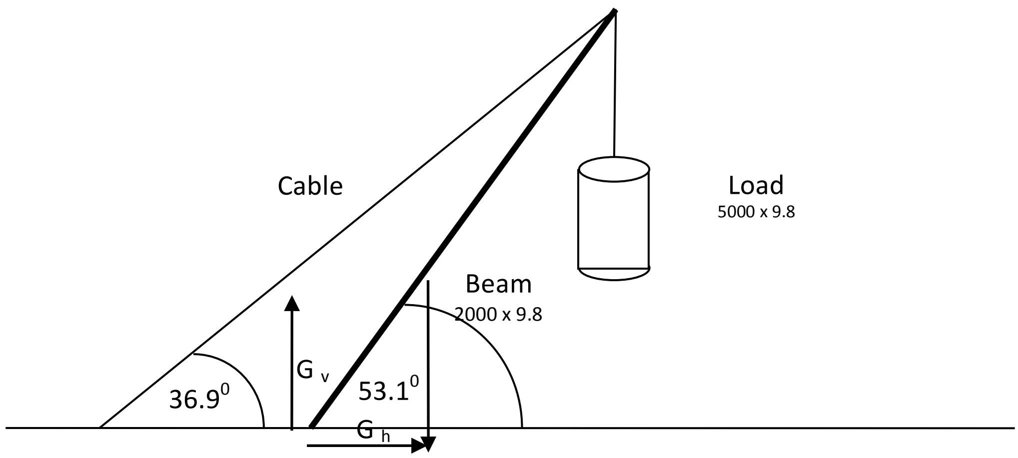
Tan θ = 147 / 156

θ = 43.3^0

Answer = A = 214 N Right 43.3^0 Up



28.



a) No dimensions are given.

Let the beam = 3 m.
(This is convenient because the COM of the beam is 1/3 rd of the way up from the ground).

Take pivots about the base of the beam, where the forces are not known

Σ Mc = Σ Ma
([1 Cos 53.1^0 x 2000 x 9.8) + (3 Cos 53.1^0 x 5000 x 9.8) = (3 x T_{perp})
1.176 x 10^4 + 8.826 x 10^4 = 3 x T_{perp}
3.334 x 10^4 = T_{perp}

Angle in the top corner is 16.2^0
Cos (73.8^0) = T_{perp} / T
Cos (73.8^0) = 3.334 x 10^4 / T
T = 1.195 x 10^5 N

b) Forces at G
Σ F up = Σ F down
G_v = (2000 x 9.8) + (5000 x 9.8) + T_v

Cos 53.1 = T_v / 1.195 x 10^5
T_v = 1.195 x 10^5 N x Cos 53.1
T_v = 7.175 x 10^4 N

G_v = (2000 x 9.8)+(5000 x 9.8)+7.175x10^4
G_v =1.404 x 10^5 N

Σ F left = Σ F right
G_h = T_h

Cos 36.9 = T_h / 1.195 x 10^5
T_h = 1.195 x 10^5x Cos 36.9
T_h = 9.556 x 10^4 N

G_h = 9.556 x 10^4 N

Combine the G_h and G_v

Pythagoras

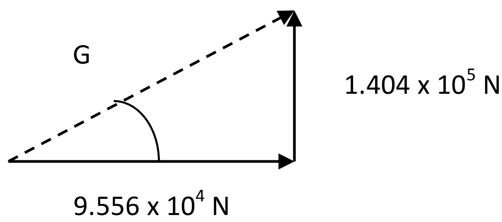
$G = (9.556 \times 10^{4\,2} + 1.404 \times 10^{5\,2})^{½}$

$G = 1.698 \times 10^5$ N

$\tan\theta = 1.404 \times 10^5 / 9.556 \times 10^4$

$\theta = 55.76^0$

Answer = G = 1.70×10^5 N Right 55.8^0 Up



29. **a) From the triangle …**

$\cos\theta = 0.3 / 0.5$

$\theta = 53.1$

Σ M clock= Σ M anti

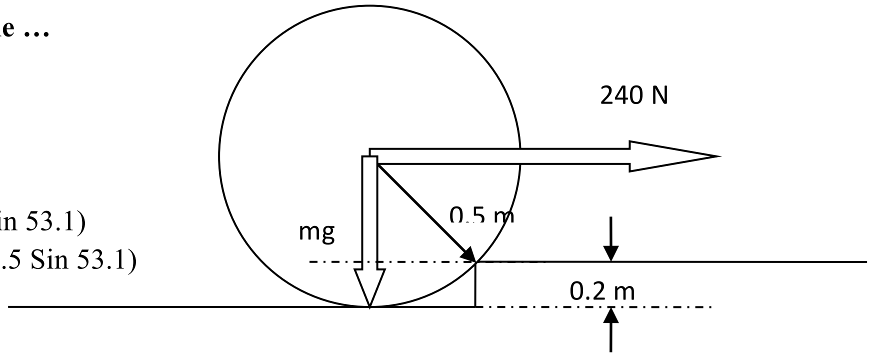
(240 x 0.3) = (mg x 0.5 Sin 53.1)

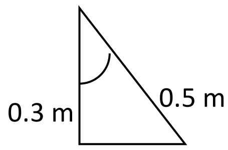
(240 x 0.3) = (m x 9.8 x 0.5 Sin 53.1)

72 = m x 3.91845

72 / 3.91845 = m

m = 18.4 kg





b)

The force applied will be minimised if it is applied at right angles to the distance

Torque applied by 240 N is …

(240 x 0.3) = 72 N m

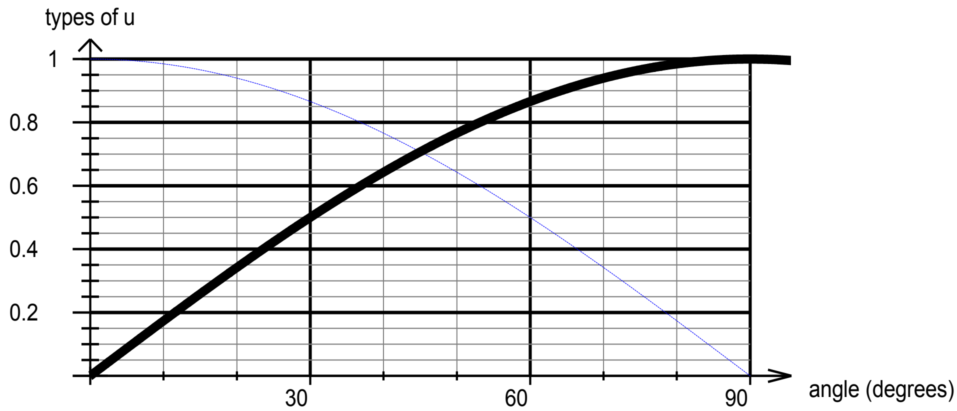
The alternative is to apply a smaller force at right angles at a distance of 0.5 m

72 = 0.5 x F at right angles

F = 144 N at 53.1 degrees above horizontal.

Problem Set 3: Projectile motion and air resistance

1. The launch angle of 45.0^0 is a compromise between maximising the vertical and horizontal components of the launch velocity. Increasing the vertical component increases the flight time before landing. Increasing the horizontal component increases the horizontal distance covered while the projectile is in the air. A launch angle of 45.0º - which only maximises the range when the take-off and landing heights are the same – optimises the interaction between these two quantities. For projectiles in which the landing height is lower than take-off, the range is maximized by using an angle slightly lower than 45.0^0. For landing heights higher than take-off height the range is maximized by using an angle slightly above 45.0^0.

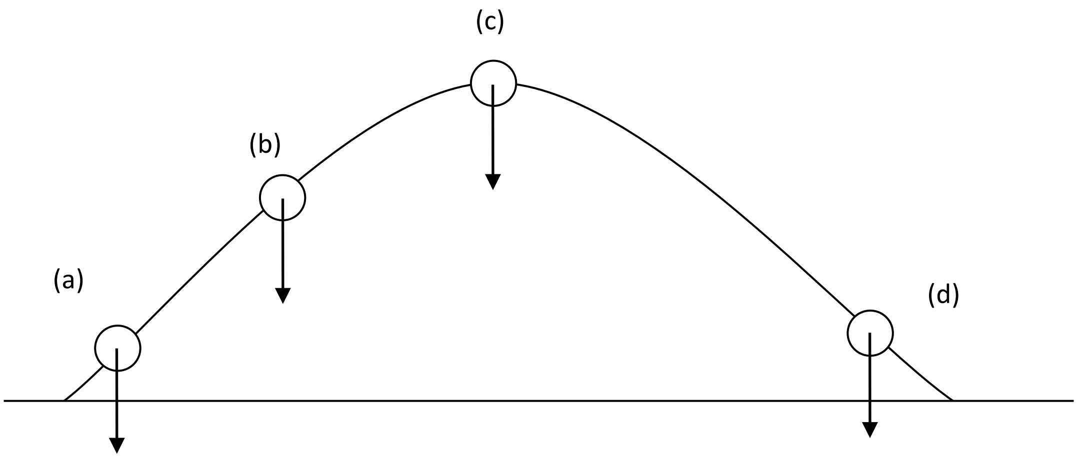


solid line = u_v = u sin (angle)
dotted line = u_h = u cos (angle)

45.0^0 is the point where the two functions are maximised. (linear programming)

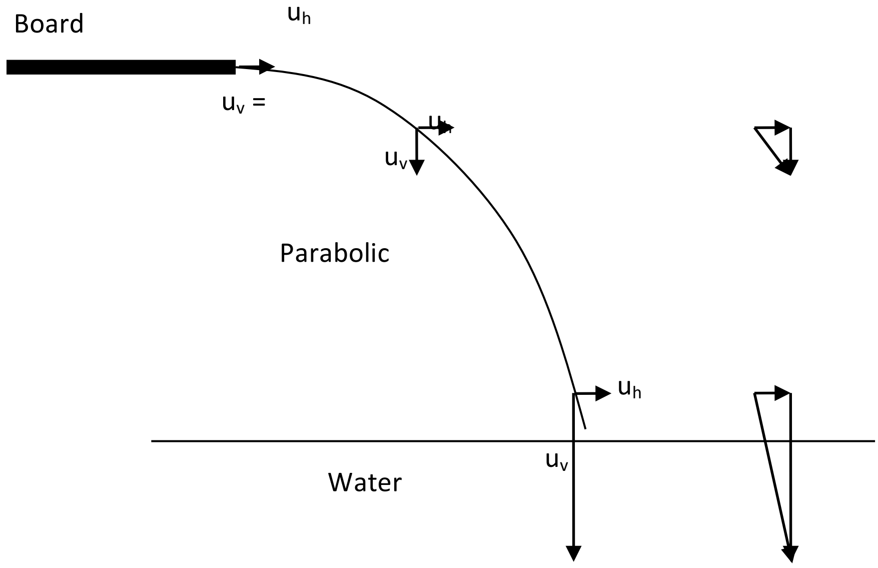
2. If the force of air resistance is ignored, then there is only one force acting on the ball – gravity. The force of gravity (weight) acts vertically downwards towards the ground – it has no horizontal component. Therefore, there is no component of the weight force that can accelerate the ball in a horizontal direction. Hence, if air resistance is ignored, the velocity of the ball in the horizontal plane will be constant.

3.



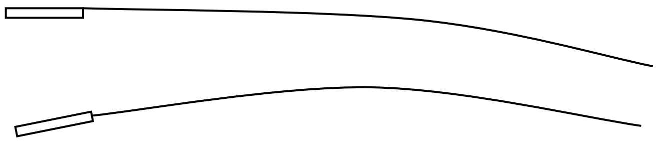
All the forces (weight of the ball) are the same size (ie, each vector should be the same length and act vertically downwards). No other force vectors should be drawn.

4.

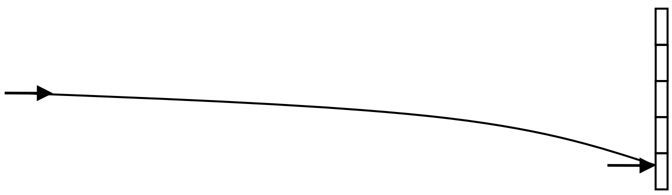


Note: u_h vectors should be the same length and horizontal in direction throughout the diver's flight.

5. Throughout its flight, the bullet is accelerated towards the ground by the force of the bullet's weight. This causes the bullet to lose height. By aiming above the target, the bullet will follow a parabolic trajectory; the bullet will rise to a maximum height and then fall back to the height at which it will hit the target.



6. This question requires that you assume the original height of the arrow from the ground and that the centre of the target is at the same height.

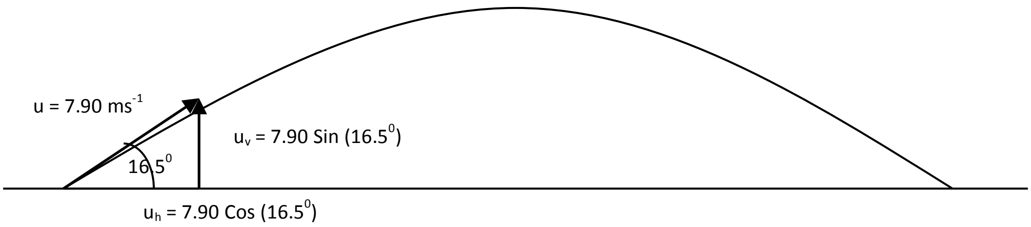


Vertical	Horizontal
$a = -9.8\ ms^{-2}$	$u_h = 80\ ms^{-1}$
$s = ?$	$s_h = 16\ m$
$s = ut + ½at^2$	$u = s / t$
$s = 0 + ½ \times -9.8 \times 0.200^2$	$t = 16 / 80$
$s = -0.196$ m	**$t = 0.200$ s**

ORANGE: r_1 = 0.0600 m; YELLOW: r_2 = 0.0600 m – 0.180 m; BROWN: r_3 = 0.180 m – 0.420 m

∴ Arrow lands in the BROWN zone = 0.016 m (scores 5 points)

7. In this question, assume that the long jumper takes off and lands at the same height



Vertical

$a = -9.8\ ms^{-2}$
$u_v = 7.90\ Sin\ (16.5^0) = 2.24\ ms^{-1}$

$u = 2.24\ ms^{-1}$; $v = -2.24\ ms^{-1}$; $a = 9.80\ ms^{-2}$

$v = u + at$
$-2.24 = +2.24 + (-9.80)\ t$
$-4.48 / -9.80 = t$

$t = 0.457\ s$

Horizontal

$u_h = 7.90\ Cos\ (16.5^0) = 7.57\ ms^{-1}$
$t = 0.457\ s$
$s_h = ?$

$7.57 = s / 0.457$
$s = 7.57 \times 0.457$

$s = 3.46\ m$

8.

	Throw as fast as possible	Throw slower
Advantages	The force with which the ball hits the tins will be larger and directed horizontally through the cans with only a small component in the vertical.	The air resistance experienced by the balls will be smaller and will be less likely to steer the ball off course, causing it to miss the target.
Disadvantages	The air resistance on the ball will be larger due to the increased velocity. This air resistance, if not evenly distributed, will cause the ball to steer off course and slow dramatically.	The force with which the ball hits the cans will be reduced; and the horizontal component of that force will also be smaller because the ball will be on a steeper trajectory downwards when it hits the cans.

9. $u_v = 4.00\ ms^{-1}$
$s = -1.10$ m
$a = -9.80\ ms^{-2}$
v= ?
t = ?

$v^2 = u^2 + 2as$
$v^2 = 4.00^2 + 2 \times -9.80 \times -1.10$
$v^2 = 16 + 21.6$
$v^2 = 37.6$
$v = -\ 6.13\ ms^{-1}$

$v = u + at$
$-\ 6.13 = 4.00 + (-9.80)t$
t = 1.03 s

10. The ball takes off and lands at a different height.

Vertical
$a = -\ 9.80\ ms^{-2}$
s = ?
$u = 22.5 \text{ Sin } 10^o =$ **$3.91\ ms^{-1}$**

$s = ut + ½\ at^2$
$s = (3.91 \times 0.874) + ½ \times -9.80 \times 0.874^2$

s = -0.326 m below the release height

1.50 + (– 0.326) **= 1.17 m above the ground.**

Horizontal
$u = 22.5 \text{ Cos } 10^o =$ **$22.2\ ms^{-1}$**
$s_h = 19.4$ m

u = s / t
t = 19.4 / 22.2
t = 0.874 s

11. a)

Vertical
$a = -\ 9.8\ ms^{-2}$
$u_v = 18 \text{ Sin } (35^0) = 10.3\ ms^{-1}$
$v_v = -10.3\ ms^{-1}$ ($s_v = 0$ m)

v = u + at
-10.3 = + 10.3 + (-9.80) t
t = -20.6 / -9.80

t = 2.10 s

b)

Horizontal
$u_h = 18 \text{ Cos } (35^0) = 14.7\ ms^{-1}$
t = 2.10 s
s_h = ?

v = s / t
14.7 = s / 2.10
s = 14.7 x 2.10

s = 30.9 m for the start

12. Assume take-off and landing heights are the same.

(25° above horizontal)

Vertical
$a = -\ 9.80\ ms^{-2}$
$u_v = 28 \text{ Sin } (25^0) = 11.8\ ms^{-1}$
$v_v = -11.8\ ms^{-1}$
t = ?

v = u + at
-11.8 = + 11.8 + (-9.80) t
-23.6 / -9.8 = t
t = 2.41 s

Horizontal
$u_h = 28 \text{ Cos } (25^0) = 25.4\ ms^{-1}$
t = 2.41 s
s_h = ?

v = s / t
s = v t
s = 25.4 x 2.41

s_h = 61.2 m

(40º above horizontal)

Vertical

$a = -9.80\ ms^{-2}$
$u_v = 28\ Sin\ (40^0) = 18.0\ ms^{-1}$
$v_v = -18.0\ ms^{-1}$

$v = u + at$
$-18.0 = 18.0 + (-9.80)\ t$
$-36.0 / -9.80 = t$

$t = 3.67\ s$

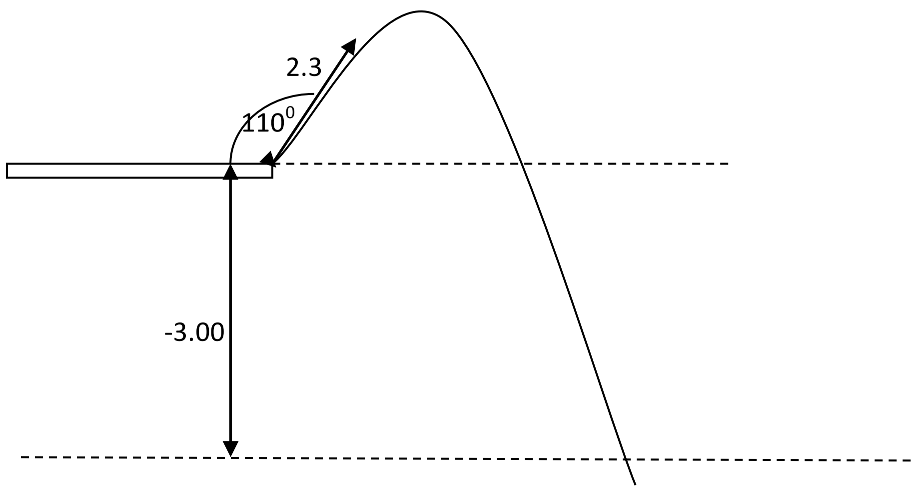
Horizontal

$u_h = 28\ Cos\ (40^0) = 21.4\ ms^{-1}$
$t = 3.67\ s$
$s_h = ?$

$v = s / t$
$s = v\ t$
$s = 21.4 \times 3.67$

$s_h = 78.5\ m$
(this will be the longest throw she can achie

13.



Vertical

$u_v = 2.30\ Sin\ 70º = 2.16\ ms^{-1}$
$s_v = -3.00\ m$
$a = -9.80\ ms^{-2}$
$t = ?$

$v^2 = u^2 + 2as$
$v^2 = 2.16^2 + 2 \times -9.80 \times -3.00$
$v^2 = 4.67 + 58.8$
$v^2 = 63.5$
$v = -\ 7.97\ ms^{-1}$

$v = u + at$
$-\ 7.97 = (2.16) + (-9.80)t$
$t = 1.03\ s$

Horizontal

$u_h = 2.30\ cos\ 70º = 0.787\ ms^{-1}$
$t = 1.03\ s$
$u_h = s / t$

$0.787 = s / 1.03$

$s_h = 0.811\ m$

14. **Vertical**

$u_v = 0\ ms^{-1}$
$s_v = ?$
$a = -9.80\ ms^{-2}$
$t = 7.62\ s$

$s_v = ut + ½\ at^2$
$s_v = 0 + ½\ (-9.80)\ (7.62)^2$
$\mathbf{s_v = -285\ m}$

Hence, water must be released 285 m above the ground.

Horizontal

$u_h = 21.0$ ms-1
$s_h = 160\ m$

$u_h = s / t$
$t = s / u_h$
$t = 160 / 21.0$

$\mathbf{t = 7.62\ s}$

15. a) **Vertical**

$u_v = 11.0 \sin 55^0 = \mathbf{9.01\ ms^{-1}}$

Horizontal

$U_h = (11.0 \cos 55^0) + 2.80$
$= 6.31 + 2.80$
$= \mathbf{9.11\ ms^{-1}}$

b) **Vertical**

$u_v = \mathbf{9.01\ ms^{-1}}$
$v_v = 0$
$a = -9.80\ ms^{-2}$
$s_v = ?$

$v^2 = u^2 + 2as$
$0^2 = 9.01^2 + 2\ (-9.80)\ s$
$\mathbf{s_v = 4.14\ m}$

$\therefore$ Max h $_{\text{above ground}} = 4.14 + 2.40$
$\mathbf{= 6.54\ m}$

Horizontal

b) **Vertical**

$u_v = 9.01\ ms^{-1}$
$a = -9.80\ ms^{-2}$
$v_v = ?$
$s_v = -2.40\ m$

$v^2 = u^2 + 2as$
$v^2 = 9.01^2 + 2(-9.80)(-2.40)$
$v_v = -11.3\ m$

$v = u + at$
$-11.3 = 9.01 + (-9.80)\ t$
$\mathbf{t = 2.07\ s}$

Horizontal

$u_h = 9.11\ ms^{-1}$
$t = 2.07\ s$

$u_h = s / t$
$s_h = u_h\ t$
$s_h = 9.11 \times 2.07$

$\mathbf{s = 18.9\ m}$

16. a) $108 / 3.6 = 30.0\ ms^{-1}$

Vertical	Horizontal
$u_v = 30.0 \text{ Sin } 15^o = \mathbf{7.76\ ms^{-1}}$	$u_h = 30.0 \text{ Cos } 15^o = \mathbf{29.0\ ms^{-1}}$

b) No. During the flight, the car will always be experiencing an acceleration of $9.80\ ms^{-2}$ vertically downwards due to gravity.

c) At its highest point (maximum height) the velocity will be $29.0\ ms^{-1}$ in a horizontal direction. At this point, the vertical component of its velocity is zero. Hence, its velocity will only consist of its original horizontal launch component (if friction is ignored).

d) Given that the jump is completed successfully with a **minimum** speed, we can assume that the car lands at D. There are a few possibilities here:

If air resistance is ignored, then the car will land at D with the same speed as at A and B. If the driver does not apply any brakes when it lands, the car will accelerate down the slope and will experience its greatest speed at E.

If air resistance is NOT ignored, then the car will land at D with a lower speed than at A and B. Again, if the driver does not apply any brakes when it lands, the car will accelerate down the slope and will experience its greatest speed at E.

In both scenarios, minimum velocity will be at C ($v_v = 0\ ms^{-1}$).

There may be other scenarios.

e) Assume that B and D are at the same height; let launch speed be 'v'.

Vertical	Horizontal
$u_v = v \text{ Sin } 15^o$	$u_h = v \cos 15^o$
$t = 39.3 / v$	$s_h = 38.0$ m
$a = -9.80$ ms-2	$u_h = s_h / t$
$s_v = 0$ m	$u_h = 38.0 / v \cos 15^o$
	$\mathbf{t = 39.3 / v}$

$s = ut + ½ at^2$

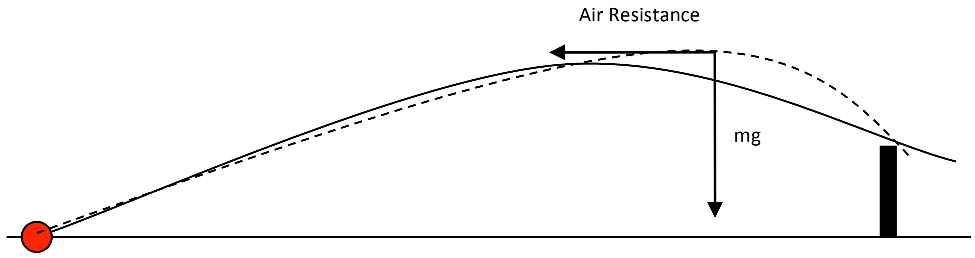
$0 = (v \text{ Sin } 15)(39.3 / v) + 0.5(-9.80)(39.3 / v)^2$

$0 = 10.2 – 7568 / v^2$

$v^2 = 742$

$v = \mathbf{27.2\ ms^{-1}}$

17. a)



b) Solid line is without air resistance.
Dotted line is with air resistance

c) **Vertical**

$u_v = 28.0 \sin 35^o = 16.06\ ms^{-1}$
$a = -9.80\ ms^{-2}$
$t = 2.79\ s$
$s_v = ?$

$s = ut + ½\ at^2$
$s = (16.06 \times 2.79) + ½\ (-9.80)\ (2.79)^2$
s = **6.67 m;** so it clears the 1.4 m high fence by a height of 5.27 m.

Horizontal

$u_h = 28.0 \cos 35^o = 22.94\ ms^{-1}$
$s_h = 64.0\ m$
$u_h = s_h / t$

$t = 64.0 / 22.94$
t = 2.79 s

The alternative way of working this question is finding out when the ball will be at a height of 1.4 m and then working backwards to determine if the range is greater than 64 m. This is the harder way of working the question. (2 possible solutions)

18. **Vertical**

$u_v = 0\ ms^{-1}$
$s_v = -(1.35 – 0.45) = - 0.90\ m$

$s = ut + ½\ at^2$
$-0.90 = (0) + ½\ (-9.80)\ (t)^2$
t = 0.429 s

Horizontal

$u_h = 83.0\ ms^{-1}$
$t = 0.429\ s$
$s_h = ?$

$u_h = s_h / t$
$s_h = u_h \times t$
$\therefore s = 83.0 \times 0.429$
$\mathbf{s_h = 35.6\ m}$

19. Let launch speed be ‘v’.

Vertical

$u_v = v \sin 48.0^o$
$s_v = 1.20\ m$
a = -9.80 ms-2
$t = 7.92 / v$

$s = ut + ½\ at^2$
$1.20 = (v \sin 48.0^o)(7.92 / v) + ½\ (-9.80)\ (7.92 / v)^2$
$1.20 = 5.89 – 307 / v^2$
$-4.69 = -307 / v^2$
$v^2 = 65.5$
$\mathbf{v = 8.09\ ms^{-1}}$

Horizontal

$u_h = v \cos 48.0^o$
$s_h = 5.30\ m$
$t = ?$
$u_h = s_h / t$

$t = 5.30 / v \cos 48.0^o$
$\therefore t = 7.92 / v$

20. **Vertical** **Horizontal**

$u_v = 13.0 \text{ Sin } 48.0^0 = 9.66 \text{ ms}^{-1}$
$v_v = 0$
$a = -9.80 \text{ ms}^{-2}$
$s_v = ?$

$v^2 = u^2 + 2as$
$0^2 = 9.66^2 + 2 (-9.80) s$
$s_v = 4.76$ m < 6.0 m (Does not hit roof)

b) **Vertical**

$u_v = 13.0 \text{ Sin } 48.0^0 = 9.66 \text{ ms}^{-1}$
$v_v = -9.66$ ms-1 (since $s_v = 0$ m)
$a = -9.80 \text{ ms}^{-2}$
$t = ?$

$v = u + at$
-9.66 = **9.66** + (-9.80) t
-19.3 = - 9.80 t
t = 1.97 s

Horizontal

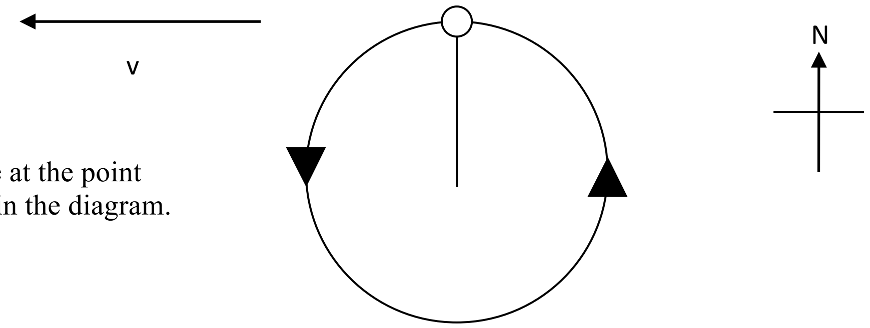
$u_h = 13.0 \text{ Cos } 48.0^0 = 8.70 \text{ ms}^{-1}$
t = 1.97 s
$s_h = ?$

$u_h = s_h / t$
$8.70 = s_h / 1.97$
$s_h = 8.70 \times 1.97$
$s_h = 17.1$ m < 18.0 m (Lands in court)

21. A long jumper becomes a projectile after they launch themselves. The objective in long jump is to maximise the horizontal displacement achieved by the projectile. Maximising launch speed will help to achieve this objective. The larger the launch speed 'v' for a particular launch angle (θ), the larger the horizontal component of this velocity (v cos θ) – which is maintained at a constant rate throughout the jump (ignoring friction). The horizontal distance achieved by the long jumper can be represented as v cos θ x t (t = flight time). Given sprinters can achieve a higher value for 'v' than other runners, they can achieve a longer jump.

Problem Set 4: Circular Motion

1. When the tangential velocity of the metal ball of the hammer is West then the ball should be released. The ball will then, only be influenced by the gravitational force.

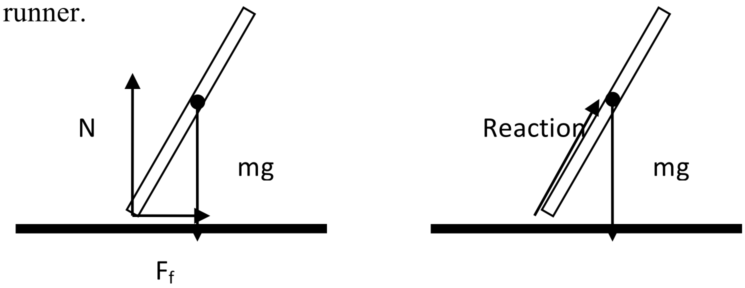


Release at the point shown in the diagram.

2. If you have only completed the projectile motion and circular motion parts of the course then the answer is ……

The sum of the forces on the runner is not equal to zero because the runner is moving in a circle.

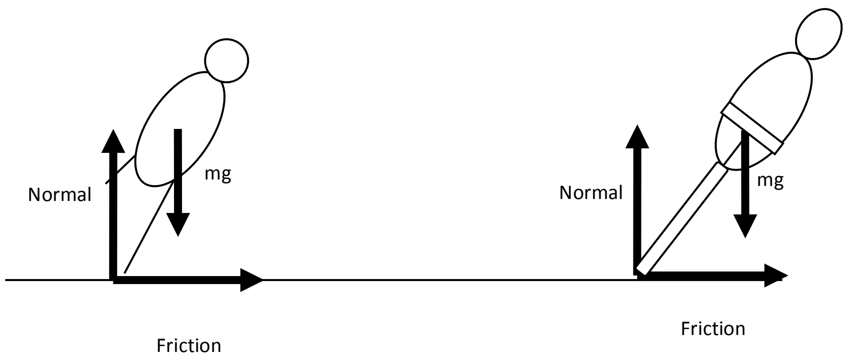
The two forces acting on the runner are mg and the reaction force. The mg force naturally passes through the centre of mass of the object because gravity acts on the centre of mass. The only force that needs to be deliberately placed through the centre of mass therefore is the reaction force. The reaction force consists of two parts or components. They are the normal force and the friction force. The angle formed when these two forces are added is the angle of lean of the runner.



If you have also completed the structures (torque) part of the course the answer is …

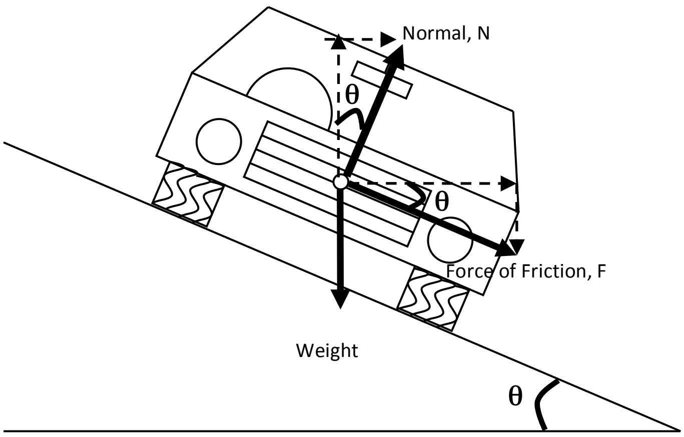
The sum of the torques on the runner is equal to zero because the runner is not spinning or toppling. In order for the torques to equal zero all of the forces need to pass through the centre of mass. If both forces (mg and reaction pass through the centre of mass then there will be no radial distance about the pivot placed at the centre of mass. The Normal force counterbalances the weight force (mg). The friction force provides the centripetal force. If the frictional force is applied without the person leaning, the person will topple as their feet take the curve and their centre of mass continues in a straight line at a tangent to the circle.

The same principle applies to bicycles rolling around a curve in a flat road.



3. The roller skater travels around the curve because of friction. The roller skater would not be able to round the cure on ice. The skater can also lean towards the turn so as not to solely rely of friction, as described in question 2.

4. Engineers make curves banked so that the normal force can contribute towards the centripetal force. If the curve was not banked, then the tyres would not be able to provided sufficient friction with the road surface to supply the centripetal force necessary to round the curve.



$N_h + F_h = F_c = mv^2/r$

Direction of turn

If the angle of banking is **q**, then F_c = N sin**q** + F cos**q** = m v^2 / r

5. $a = v^2 / r$

$a = 3.5^2 / 15$

a = 0.817 ms^{-2} towards the centre of the circle.

6. $F_c = m\ v^2 / r$

$F_c = 0.585 \times 11.5^2 / 1.25$

F_c = 61.9 N towards the centre of the circle.

7.a) T = 15.5 s and r = 3.80 m
v = s / t

$v = 2\pi r / T$

$v = 2\pi \times 3.8 / 15.5$

v = 1.54 ms^{-1}

b) $F_c = m\ v^2 / r$

$F_c = 28 \times 1.54^2 / 3.8$

F_c = 17.5 N towards the centre of the circle.

8. Refer to question 4 for an appropriate diagram

Vertical

$\Sigma F_v = 0$

$N_v + -F_v + -mg = 0$

Let the force of friction = 0

$N \cos\theta = mg$

$N = mg / \cos\theta$ → Sub into horizontal.

Horizontal

$\Sigma F_h = F_c$

$N_h + F_h = mv^2 / r$

Let the force of friction = 0

$N \sin\theta = mv^2 / r$ (insert vertical expression)

$$\frac{mg \sin\theta}{\cos\theta} = \frac{mv^2}{r}$$

$$mg \tan\theta = \frac{mv^2}{r}$$

$$\tan\theta = \frac{mv^2}{mg\, r}$$

$$\tan\theta = \frac{v^2}{g\, r}$$

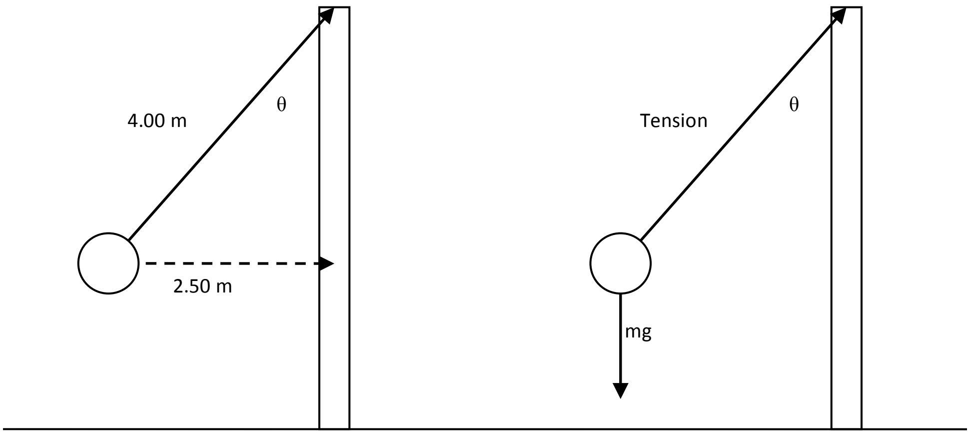
$$\theta = \tan^{-1} \frac{v^2}{g\, r}$$

$$\theta = \tan^{-1} \frac{(110/3.6)^2}{9.8 \times 300}$$

$\boldsymbol{\theta = 17.6^0}$

This result can be used wherever there is no friction on a banked bend. The angle is the angle formed between the ground and the curve – the angle of banking!

9.



a) $\sin\theta = 2.5 / 4$

gives $\boldsymbol{\theta = 38.7^0}$

b) **Vertical** | **Horizontal**

Vertical	**Horizontal**
$\Sigma F_v = 0$	$\Sigma F_h = F_c$
$T \cos\theta + -mg = 0$	$T \sin 38.7 = mv^2 / r$
$T \cos\theta = mg$	$691 \sin 38.7 \times 2.5 / 55 = v^2$
$T = 55 \times 9.8 / \cos 38.7$	$v = 4.43$ m/s
T = 691 N	$v = 2\pi r / t$
	$t = 2\pi r / v$
	$t = 2\pi\, 2.5 / 4.43$
	t = 3.55 s

10. a) Yes the car is accelerating because it is **changing direction** (moving in a circle).

b) $F_c = m v^2 / r$

$F_c = 1250\,(24/3.6)^2 / 18$

$F_c = 1250\,(6.666)^2 / 18$

$\mathbf{F_c = 3.09 \times 10^3\ N}$ **(3.09 kN)**

c) $\theta = \tan^{-1} \dfrac{(6.6666)^2}{9.8 \times 18}$ (see question 8 for derivation of this equation)

gives $\boldsymbol{\theta = 14.1^0}$

11. $\tan\theta = \dfrac{v^2}{g \times r}$ (see question 8 for derivation of this equation)

$\tan 20 = \dfrac{v^2}{(9.8 \times 70)}$

$9.8 \times 70 \times \tan 20 = v^2 = 249.7\ m^2s^{-2}$

gives $\mathbf{v = 15.8\ ms^{-1}}$

12. a) The minimum speed will occur at the shorter radius (9.5 cm)

frequency, f = 3800 RPM = 3800 / 60 = 63.33 Hz

Time period, T = 1 / f therefore f = 1 / T

$v = 2\pi r / T = 2\pi rf$

$v = 2\,\pi\, 9.5\times10^{-2} \times 63.33$

gives $\mathbf{v = 37.8\ ms^{-1}}$

b) The maximum centripetal acceleration occurs at the larger radius. If you are unsure your will need to calculate both.

$v = 2\pi r / T = 2\pi rf$

$v = 2\ \pi\ 12.0 \times 10^{-2} \times 63.33$

$v = 47.76$ m/s

$a = v^2 / r$

$a = 47.75^2 / 12 \times 10^{-2}$

gives $a = 1.90 \times 10^4$ ms^{-2}

c) The maximum force will be experienced at the 12×10^{-2} m

$F_c = m\ v^2 / r = m(2\pi rf)^2 / r$

$8.2 \times 10^{-3} = (98 \times 10^{-9} \times 10^{-3}) \times 4\ \pi^2\ 12 \times 10^{-2} \times f^2$

$f^2 = 1.77 \times 10^7\ \text{Hz}^2$ **gives $f = 4.20 \times 10^3$ Hz**

13.

a) Force is towards the centre of the circle.

b) $F_c = m\ v^2 / r$

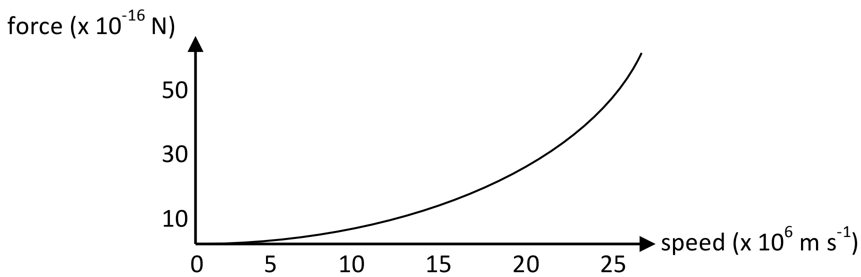
$F_c = 1.7 \times 10^{-27} \times (7.8 \times 10^6)^2 / 200$

$F_c = 5.17 \times 10^{-16}$ N **(if 1.67×10^{-27} kg is used for the mass of a proton, then answer for $F_c = 5.08 \times 10^{-16}$ N)**

c) This is a quadratic relationship, since $F_c = mv^2/r$, therefore $F_c\ a^{al}\ v^2$

So, **doubling** the velocity would require a force **four** times greater to maintain the path. Therefore, if the force is 5.2×10^{-16} N when the velocity is 7.8×10^6 m s^{-1}, then the force will be about 21×10^{-16} N when the velocity is doubled (about 16×10^6 ms^{-1}) and the force will be about 1.3×10^{-16} N (a quarter of the original force) when the velocity is halved (about 4×10^6 ms^{-1}).

The graph would therefore be of the form, $y = x^2$



d) The time period in question should be 2.50 s (and not 2.00 s as quoted in this part of the question. The proton is accelerating and therefore it is completing each revolution quicker and quicker as the time progresses and it is not as simple as multiplying the 440,000 by 0.8 (2.0 ÷ 2.5) to try to obtain the new number of revolutions).

$s = 2\ \pi\ 200 \times 440\ 000 =$ **5.53×10^8 m (553 Mm)**

e) The proton will fall due to the effects of gravity. Assume that the proton has no vertical velocity on entering the synchrotron.

$s = ut + \frac{1}{2} at^2$

$s = 0 + \frac{1}{2} (9.8) \times 2.5^2$

s = 30.6 m down (a significant drop if not taken into account)

f) A field (electric or magnetic) is required to provide an upward force to counterbalance the protons weight.

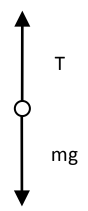
In order to keep the proton circling horizontally, a vertical force must be applied to the proton in order to oppose its weight and hence prevent it losing height.

Opposing force required = weight of proton,
$w = m \times g = 1.7 \times 10^{-27}$ kg $\times$ 9.8 ms^{-2} = 1.67×10^{-26} N upwards.
This could be provided by an appropriate magnetic field or electric field.

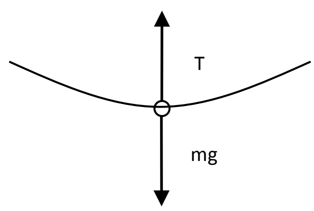
14.

a) The tension in the string provides the centripetal force as well as supporting the objects weight.

Object hanging from string (not swinging)



Object swinging from string



$mg + T = 0$

$T = -mg$ (forces are equal, but opposite)

$T - mg = mv^2 / r$

$T = mv^2 / r + mg$

Based on the equations above, the tension is greater in the string when the object is swinging because the tension and the centripetal force add and therefore the string is likely to snap.

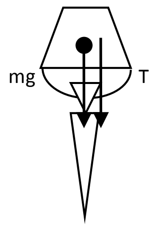
15a) This answer is based on an estimated radius of 0.8 m

The assumption built into the wording of the question is that the "other" force (arm tension of compression) is set to 0.

$\Sigma F_v = mv^2 / r$

$T + mg = mv^2/r$

Let $T = 0$



$mg = mv^2/r$

$g = v^2/r$

$v = \sqrt{gr}$

$v = \sqrt{(9.8 \times 0.8)}$

$v = 2.8\ ms^{-1}$ (about $3\ ms^{-1}$)

b) The bucket is being 'driven' towards the centre of its circular path due to the presence of a resultant force providing a centripetal force. The water contents of the bucket, is maintaining its inertia (Newton's First Law) and would 'feel' like it is being forced towards the outside of the circular path (the misconceived 'centrifugal' force). Hence, the water remains in the bucket.

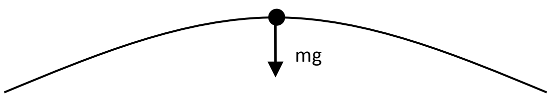
This is the same as the effect you feel as a passenger in a car which is turning a corner (much more noticeable if the car is taking the corner at speed!) – the car is being 'pulled' towards the centre of the turning circle and you feel like you are being pushed outwards, when in reality you are just feeling the car being 'pulled' inwards.

c) The bucket can travel at a constant minimum speed around its circular path, however the arm muscles would have to work a bit harder at different points of its path – the highest and lowest points of its vertical path would be the two extremes. If the arm muscles applied a constant tension force, then the speed would have to be regularly increased and decreased in order to maintain a vertical circular path.

There is also a 'loss' in potential energy from the top of the path towards the bottom, which would suggest that, since the total amount of energy must remain constant, then there must be a gain in kinetic energy (and therefore speed) in order to compensate for this loss.

16.
a) r = 800m

The other force (reaction force) (lift) on the pilot is 0.



$\Sigma F_v = mv^2/r$

since lift = 0, then: $mg = mv^2/r$

$mg = mv^2/r$

$g = v^2/r$

$v = \sqrt{gr}$

$v = \sqrt{(9.8 \times 800)}$

$v = 88.5\ ms^{-1}$

b) At the bottom of the loop the plane has lost height, lost E_p (mgDh) and so has gained E_k ($\frac{1}{2}mv^2$). The total amount of energy, E_{total} must remain constant, so $E_{total} = E_p + E_k$

Conservation of energy

E_{total} at top = E_{total} at bottom

$Dh = 2r = 2 \times 800 = 1600$ m (relative to bottom)

E_k at top + E_p at top = E_k at bottom

$\frac{1}{2} m u^2 + mgDh = \frac{1}{2} m v^2$

$\frac{1}{2} u^2 + gDh = \frac{1}{2} v^2$

$\frac{1}{2} \times 88.5^2 + 9.8 \times 1600 = \frac{1}{2} v^2$

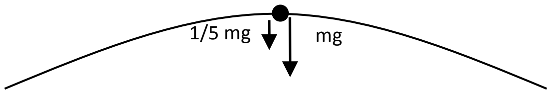
$3916 + 15680 = \frac{1}{2} v^2$

$19596 = \frac{1}{2} v^2$

$39192 = v^2$

gives v = 198 ms^{-1}

17. Two forces act on the pilot, her weight (mg) and the reaction force from her seat (⅕mg) downwards.



$\Sigma F_v = mv^2 /r$

$\frac{1}{5}mg + mg = mv^2/r$

$6/5\ mg = mv^2/r$

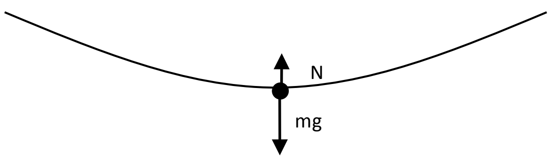
$6/5\ g = v^2/r$

$v = \sqrt{(6/5\ g\ r)}$

$v = \sqrt{(1.2 \times 9.8 \times 650)}$

gives v = 87.4 ms^{-1}

18. a)



$\Sigma F_v = mv^2 /r$

$N - mg = mv^2/r$

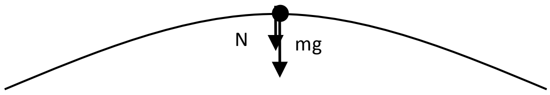
$N = mv^2/r + mg$

$N = (2 \times 20^2 / 5) + (2 \times 9.8)$

$N = 160 + 19.6$

gives N = 180 N upwards (to 3 sig. figs.)

b)



$$\Sigma F_v = mv^2/r$$

$N + mg = mv^2/r$

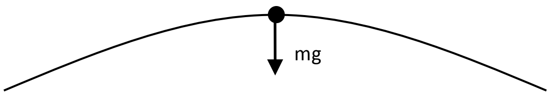
$N = mv^2/r - mg$

$N = (2 \times 10^2 / 5) - (2 \times 9.8)$

$N = 40 - 19.6$

gives N = 20.4 N downwards

19. a)



$$\Sigma F_v = mv^2/r$$

'Lift' force = 0, since you feel 'weightless'

$mg = mv^2/r$

$g = v^2/r$

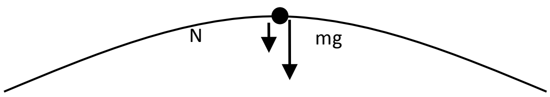
$r = v^2/g$

$r = 14^2 / 9.8$

gives r = 20.0 m

b) If the roller coaster travels faster than 14.0 ms^{-1} more force will need to be supplied downwards to assist the mg force in providing the resultant centripetal force necessary to keep the carriage in the loop. This extra force will come from the normal force (reaction force) of the tracks in a downward direction.

For example, let $v = 20\ ms^{-1}$



$$\Sigma F_v = mv^2/r$$

$N + mg = mv^2/r$

$N = m(v^2/r - g)$

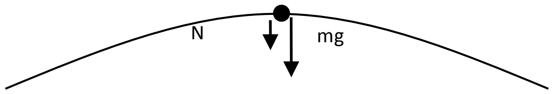
$N = m(20^2/20 - 9.8)$

$N = (20 - 9.8)\ m$ (i.e. just over 10 times your mass)

N = (10.2 m) N in the direction shown in the free body diagram

c) If the roller coaster travels slower than 14.0 ms^{-1} less force will need to be supplied downwards. A force will need to be provided upwards to reduce the effects of the mg force. This extra force upwards needs to come from the track mechanism, which depending on the track design may actually be physically impossible (if no safety devices).

For example, let v = 10 ms^{-1}



$\Sigma F_v = mv^2 /r$

$N + mg = mv^2/r$

$N = mv^2/r - mg$

$N = m (v^2/r - g)$

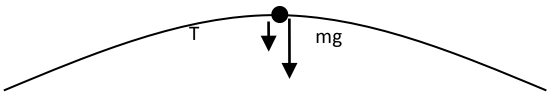
$N = m (10^2/20 - 9.8)$

$N = (5 - 9.8)$ m (i.e. just under 5 times your mass)

$N = (-4.8 m)$ N

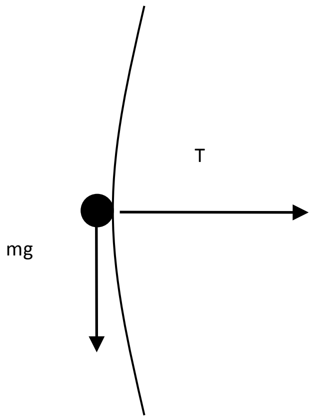
N = (4.8 m) N in the <u>opposite</u> direction to that shown in the free body diagram – upwards!

20. a)



where T is the tension force created by her arm muscles / bones / upper body.

b)



Conservation of energy

E_{total} at top = E_{total} at bottom = E_{total} at middle

$Dh_{top} = 2\ r = 1.8$ m (relative to bottom)

$Dh_{middle} = r = 0.9$ m (when level with bar / relative to bottom)

E_k at top + E_p at top = E_k at middle + E_p at middle

$\frac{1}{2} m u^2 + mgDh_{top} = \frac{1}{2} m v^2 + mgDh_{middle}$

$\frac{1}{2} u^2 + gDh_{top} = \frac{1}{2} v^2 + gDh_{middle}$

$\frac{1}{2} \times 1^2 + 9.8 \times 1.8 = \frac{1}{2} v^2 + 9.8 \times 0.9$

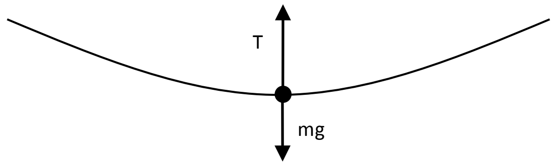
$\frac{1}{2} + 17.64 = \frac{1}{2} v^2 + 8.82$

$18.14 - 8.82 = 9.32 = \frac{1}{2} v^2$

$18.64 = v^2$

gives v = 4.32 ms^{-1} directly downwards

c)



Conservation of energy

E_{total} at top = E_{total} at bottom

$Dh_{top} = 2\ r = 1.8$ m (relative to bottom)

E_k at top + E_p at top = E_k at bottom
(since relative E_p at bottom = 0)

$\frac{1}{2}\ m\ u^2 + mgDh_{top} = \frac{1}{2}\ m\ v^2$

$\frac{1}{2}\ u^2 + gDh_{top} = \frac{1}{2}\ v^2$

$\frac{1}{2} \times 1^2 + 9.8 \times 1.8 = \frac{1}{2}\ v^2$

$\frac{1}{2} + 17.64 = \frac{1}{2}\ v^2$

$18.14 = \frac{1}{2}\ v^2 \quad \therefore \quad 36.28 = v^2$

gives $v = 6.02\ ms^{-1}$

$\Sigma\ F_v = mv^2 /r$

$T - mg = mv^2/r$

$T = mv^2/r + mg$

$T = (40 \times 6.02^2 / 0.9) + (40 \times 9.8)$

$T = 1611 + 392$

T = 2003 N upwards

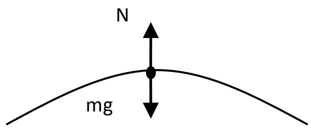
(2.00 kN upwards, provided by a tension force created by her arm muscles / bones / upper body)

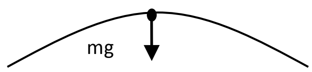
21. a)

LOW

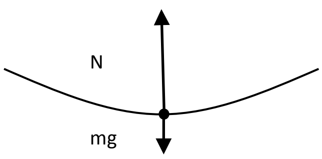
HIGH

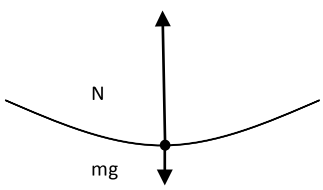
Top





Bottom





b) Based on feeling 'weightless' on the HIGH setting, the speed, v of the Ferris wheel is: (refer to diagram 'top right')

$mg = mv^2 / r$

$g = v^2 / r$

$v = \sqrt{(r\ g)}$

$v = \sqrt{(3.6 \times 9.8)}$

gives $v = 5.94\ ms^{-1}$

c) On the **HIGH** setting:

At the top, reaction force = 0 due to the feeling of 'weightlessness'

However, at the bottom (refer to diagram 'bottom right'):

$N - mg = mv^2 / r \therefore N = mv^2 / r + mg$ (passengers revolve at constant speed)

$N = (60 \times 5.94^2 / 3.6) + (60 \times 9.8) \therefore N = 588 + 588$

gives reaction force, N = 1176 N (1.18 kN)

d) On the **LOW** setting (speed is half the HIGH speed = 5.94 / 2 = 2.97 ms^{-1}):

At the top (refer to diagram 'top left'):
$mg - N = mv^2 / r$
$N = mg - mv^2 / r$
$N = (60 \times 9.8) - (60 \times 2.97^2 / 3.6) \therefore N = 588 - 147$
gives reaction force, N = 441 N

At the bottom (refer to diagram 'bottom left'):

$N - mg = mv^2 / r$
$N = mv^2 / r + mg$
$N = (60 \times 2.97^2 / 3.6) + (60 \times 9.8) \therefore N = 588 + 147$
gives reaction force, N = 735 N

22.

a) E_{total} at top = E_{total} at bottom = E_{total} at middle

$Dh_{point\ A} = 2\ r = 4.00$ m (relative to bottom of swing)

$Dh_{point\ X} = r = 2.00$ m (relative to bottom of swing)

$Dh_{point\ B} = 0$ (relative to bottom of swing)

Consider the situation of the stone at Point A (highest point of its swing):

E_k at point A + E_p at point A = E_k at point X + E_p at point X

$\frac{1}{2}\ m\ u_A^2 + mgDh_{point\ A} = \frac{1}{2}\ m\ V_X^2 + mgDh_{point\ X}$

$\frac{1}{2}\ u_A^2 + gDh_{point\ A} = \frac{1}{2}\ v^2 + gDh_{point\ X}$

$\frac{1}{2} \times u_A^2 + (9.8 \times 4) = \frac{1}{2} \times 10.4^2 + (9.8 \times 2)$

$\frac{1}{2} \times u_A^2 + 39.2 = 54.08 + 19.6 = 73.68$

$73.68 - 39.2 = 34.48 = \frac{1}{2} \times u_A^2$

$68.96 = u_A^2$

gives $u_A = 8.30\ ms^{-1}$

Now, consider the situation of the stone at Point B (lowest point of its swing):

E_k at point A + E_p at point A = E_k at point B + E_p at point B

$\frac{1}{2}\ m\ u_A^2 + mgDh_{point\ A} = \frac{1}{2}\ m\ V_B^2 + mgDh_{point\ B}$

$\frac{1}{2}\ u_A^2 + gDh_{point\ A} = \frac{1}{2}\ V_B^2 + gDh_{point\ B} + 0$

$\frac{1}{2} \times 8.3^2 + (9.8 \times 4) = \frac{1}{2}\ V_B^2$

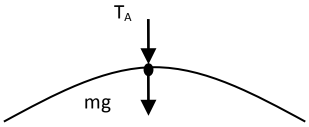
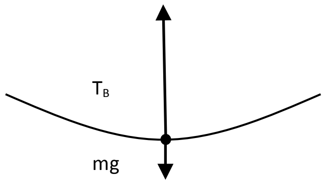
$34.45 + 39.2 = \frac{1}{2}\ V_B^2$

$73.65 = \frac{1}{2}\ V_B^2$

$147.3 = V_B^2$

gives $V_B = 12.1\ ms^{-1}$

22. b)

Top of swing (i.e. at point A)	Bottom of swing (i.e. at point B)
	

Tension at the top = T_A

$mg + T_A = mv^2 / r$

$T_A = mv^2 /r - mg$

$T_A = (2.5 \times 8.3^2 / 2) - (2.5 \times 9.8)$

$T_A = 86.11 - 24.5$

gives T_A = 61.6 N downwards

Tension at the bottom = T_B

$T_B - mg = mv^2 / r$

$T_B = mv^2 /r + mg$

$T_B = (2.5 \times 12.1^2 / 2) + (2.5 \times 9.8)$

$T_B = 183 + 24.5 = 207.5$

gives T_B = 208 N upwards (3 sig. figs.)

c) The string is most likely to break **at the bottom** (**point B**), since the tension is greater here. The tension must counterbalance weight and also provide the centripetal force. Mathematical verification:

Tension at top (point A), T_A	**Tension when stone is horizontal (at point X), T_X**	**Tension at bottom (point B), T_B**
	$T_X = mV_X^2 / r = (2.5 \times 10.4^2 / 2)$	
61.6 N	(since 'mg' has no effect horizontally	**208 N**
(as given above)	**gives T_X = 135 N**	(as given above)

Set 5: Gravitation and Satellites

1. You are attracted to large buildings. Unfortunately the mass of yourself and the large building is not large enough to create a measureable force.

   For example, assume that a 100 kg man is standing 1 m from a building of mass 10,000 tonnes (10 million kg).

   The force of attraction between the building and this man, $F = G\ M_{building}\ M_{man} / r^2$

   $F = 6.67 \times 10^{-11} \times 1 \times 10^{7} \times 100 / 1^2 = 6.67 \times 10^{-2}$ N

   (friction between the ground and feet of the man would be significantly greater)

2. a) Yes their weight does change but not by much. The increase in radial distance from the centre of the planet is not large enough to produce a measurable effect or an effect that is noticeable to the brain of the climber.

   Since $g = GM_{earth} / r^2$ then $g\ \alpha^{al}\ 1/r^2$ (where r = distance from centre of Earth)

   On Earth's surface, $r = 6.37 \times 10^6$ m

   On Mount Everest, $r = (6.37 \times 10^6 + 8000) = 6.378 \times 10^6$ m, therefore their weight would **decrease** by a very small amount due to this slightly increased distance.

   b) The weight of an underground miner decreases as they descend into a mine, though the effect is not easily measured. The reason that this occurs is that as you descend into the Earth, the Earth above you attracts you upwards slightly while the Earth below you continues to pull you down, though to a slightly reduced extent. If you were able to descend to the centre of the Earth you would eventually become weightless because you would have equal quantities of matter all around you, pulling you equally in all directions. The universal gravitational law does not operate below the surface of the Earth. Instead the gravitational field drops to zero linearly as you move from the surface of the Earth to its centre.

   c) The density of the Earth is not uniform. More dense rocks (rocks that have more mass per unit of volume they take up) in the crust will give a stronger gravitational reading than lighter rocks. Also, the Earth is not perfectly spherical and its radius differs at different locations around its surface, leading to higher readings of the acceleration due to gravity where the radius is smaller.

3. The object has weight (mg). The object appears "weightless" as it begins to fall because there are no other forces acting on it. The perception (feeling) of weight is due the presence of other forces such as normal force or air resistance opposing the weight force. The falling object will continue to "feel" weightless until it approaches terminal velocity and the air resistance force becomes appreciable. The air resistance force will then provide the "other" force that will allow you to "feel" your weight.

4. The acceleration of an object is not determined by the mass of the object. It is determined by the mass of the planet (other objects) that is making (generating) the gravitational field.

   In the formula for gravitational filed strength created by a planet, $g = GM_p / r^2$ the mass of the object is not listed and therefore irrelevant. Only the mass of the planet and the distance from the centre of the planet (field), are listed.

5. The formula for the force, F acting on an object in the gravitational field of a star is:

$F = G\, M_{object}\, M_{star} / r^2$. This formula can only be applied outside of the star's surface. Inside the star, a different formula holds true. If the volume of the star is reduced however without altering the stars mass, the edge of the star becomes closer to the centre of the star and the formula above will still hold true for longer until you pass below the star's surface.

Decreasing the volume of an object without altering its mass increases the density of the object ($\rho = M / V$ rearranged, $M = \rho\uparrow V\downarrow$).

In the formula $F = G\, M_{object}\, M_{star} / r^2$, as r decreases, the size of the force (and consequently gravitational field) increases. If r is small enough without going inside the stars surface, the strength of gravitational field will be so great that not even light can escape.

6. $g = G\, M / r^2$

$10 = 6.67 \times 10^{-11} \times M_{earth} / (6.37 \times 10^6)^2$

$M = 6.08 \times 10^{24}$ kg

7. $F = G\, M_1\, M_2 / r^2$

$F = 6.67 \times 10^{-11} \times 100 \times 100 / 0.622^2$

$F = 1.724 \times 10^{-6}$ N

8. a)

On surface

$F = G\, M_{earth}\, M_{shuttle} / r^2$ ------- (1)

Above surface

$\frac{1}{2} F = G\, M_{earth}\, M_{shuttle} / X^2$ ---------- (2)

Substitute F on surface (1) into F above surface (2)

$$\frac{\frac{1}{2} G\, M_{earth}\, M_{shuttle}}{r^2} = \frac{G\, M_{earth}\, M_{shuttle}}{X^2}$$

Cancelling common terms gives:

$$\frac{\frac{1}{2}}{r^2} = \frac{1}{X^2}$$

therefore $\frac{1}{2} X^2 = r^2$ so $X^2 = 2\, r^2$ and $\mathbf{X = \sqrt{2}r}$

if $r = 6.37 \times 10^6$ m, then $X = 9.01 \times 10^6$ m

Height above the surface will be $(X - R_{earth}) = (9.01 \times 10^7 - 6.37 \times 10^6) = h$

gives **$h = 2.64 \times 10^6$ m**

8. b) $g = G\ M_{earth} / r^2$

$g = 6.67 \times 10^{-11} \times 5.97 \times 10^{24} / (6.37 \times 10^6 + 610 \times 10^3)^2$

$g = 6.67 \times 10^{-11} \times 5.97 \times 10^{24} / (6.98 \times 10^6)^2$

$g = 8.17\ m/s^2$ towards the earth

$F_{gravitaitonal} = F_{centripetal}$

$$\frac{G\ M_{earth}\ M_{telescope}}{r^2} = \frac{M_{telescope}\ v^2}{r}$$

$$\frac{G\ M_{earth}}{r} = v^2$$

c) $$v^2 = \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{6.98 \times 10^6} = 5.70 \times 10^7\ m^2s^{-2}$$

$v = 7.55 \times 10^3\ m/s$ ($7.55\ kms^{-1}$)

9. $F = G\ M_{earth}\ M_{moon} / r^2$

$2.03 \times 10^{20} = 6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 7.35 \times 10^{22} / r^2$

$r^2 = 1.44 \times 10^{17}\ m^2$

$r = 3.80 \times 10^8\ m$

10.

g on surface of Earth

$g_e = G\ M_e / r_e^2$

g on surface of Neptune

$$g_n = \frac{G\ 16.6M_e}{(3.89r_e)^2}$$

$$g_n = \frac{G\ M_e \times 16.6}{(r_e \times 3.89)^2}$$

$$g_n = \frac{G\ M_e \times 16.6}{r_e^2 \times 15.1321}$$

$$g_n = \frac{G\ M_e \times 1.097}{r_e^{\ 2}}$$

But $g_e = G\ M_e / r_e^2$ substituting in we get

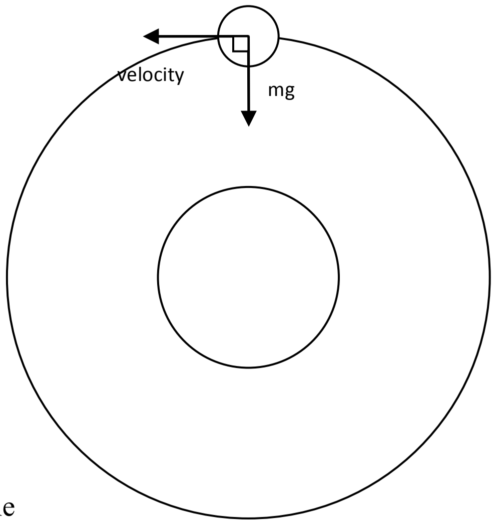
$g_n = g_e \times 1.097$

The ratio $g_e : g_n$ is 1:1.10

(If $g_e = 9.80$ then $g_n = 9.80 \times 1.097 =$ **$10.8\ m/s^2$**)

11. a) The gravitational force of the earth acts at right angles to the velocity of the moon. The acceleration of the earth's gravity acting on the mass of the moon (moons weight) has no component that is in the same direction as the velocity of the moon and so cannot assist it to alter its velocity.

Note :- This is not a free body diagram because free body diagrams **only** list the forces acting on the object. Velocity is not a force.



Also:

$$F_{gravitaitonal} = F_{centripetal}$$

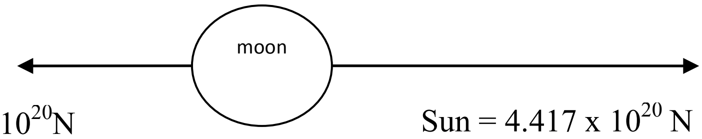
$$\frac{G\, M_{earth}\, M_{moon}}{r^2} = \frac{M_{moon}\, v^2}{r}$$

$$\frac{G\, M_{earth}}{r} = v^2$$

Therefore only the Earth's mass and the distance the moon is from the Earth determine the moon's speed; since both are constant, then the moon's speed is unchanged.

b) During a solar eclipse the Earth, moon and sun are all in a line. The total force on the moon will be the vector sum of the force effects of the earth and the sun.

F of Earth on Moon	**F of Sun on Moon**
$F_e = G\, M_{earth}\, M_{moon} / r_{em}^2$	$F_s = G\, M_{sun}\, M_{moon} / r_{sm}^2$
$F_e = \dfrac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 7.35 \times 10^{22}}{(3.8 \times 10^8)^2}$	$F_s = \dfrac{6.67 \times 10^{-11} \times 1.99 \times 10^{30} \times 7.35 \times 10^{22}}{(1.49 \times 10^{11} - 3.8 \times 10^8)^2}$
$F_e = 2.027 \times 10^{20}$ N	$F_s = \dfrac{6.67 \times 10^{-11} \times 1.99 \times 10^{30} \times 7.35 \times 10^{22}}{(1.4862 \times 10^{11})^2}$
	$F_s = 4.417 \times 10^{20}$ N



Earth = 2.027×10^{20}N   Sun = 4.417×10^{20} N

Net Force = $4.417 \times 10^{20} - 2.027 \times 10^{20}$

Net Force = 2.39×10^{20} N towards the sun

12. The greater the distance the longer the period.

According to Kepler's law $\frac{r^3}{T^2} = \frac{G\, m_p}{4\,\pi^2}$ as r gets bigger T gets bigger also since $\frac{G\, m_p}{4\,\pi^2}$ is constant. So, $T^2\ \alpha^{al}\ r^3$, therefore a satellite orbiting at a height in excess of 30,000 km will have the greater time period.

13. a) The satellite's orbit would change from circular to elliptical. The balance between the speed of the satellite and the centripetal force has been broken. If a constant retarding force is encountered such (as the Earth's atmosphere), the pathway taken by the satellite will become an inward spiral (death spiral).

b) The satellite's period of revolution is matched to the period of the Earth's rotation. This is achieved using Kepler's law $\frac{r^3}{T^2} = \frac{G\, m_p}{4\,\pi^2}$ to calculate the correct distance above the earth to park the satellite in order to achieve an orbital period equal to 24 hours. The satellite does not fall back to earth because its rate of acceleration towards the Earth, is counterbalanced by its rate movement (velocity) away from the Earth at a tangent.

14. a) To move a satellite into orbit it is necessary to increase the gravitational potential energy of the satellite. This is done by giving the satellite kinetic energy, usually provided by a rocket. The calculation according to the law of conservation of energy begins to look like:

E_p (at Earth's radius, r_{earth}) + E_k (rocket) = E_p (at r_{earth} + altitude)

An object on the equator already has some kinetic energy due to the revolution of objects on the Earth's surface as compared to an object at a geographic pole (just rotating). This means a satellite on the equator needs less supplementary (extra) kinetic energy to get it into orbit. This is why the USA's space agency is in Florida on the equator.

The Earth rotates towards the East. If the rocket is shot in the opposite direction to the rotation of the Earth (west) you are actually removing the kinetic energy that the Earth has given to the satellite. If the rocket is shot in the same direction to the rotation of the Earth (west) you are adding to the kinetic energy that the Earth has given to the satellite, almost like a catapult effect. This is what you want.

b) See 14. a) above.
The speed of rotation of the earth is at a maximum at the equator and so the kinetic energy given to the rocket / satellite by the Earth's rotation is also at a maximum. It will require less rocket fuel to make up the extra kinetic energy required to get the satellite into orbit.

15. Reduce the orbiting speed of the capsule so the capsule goes into an elliptical orbit, bringing it in contact with the Earth's atmosphere resulting in an inward spiral to Earth.

16. According to the law of conservation of energy, the gravitational potential energy is being converted into kinetic energy as the satellite descends.

When you use the more general form of the gravitational potential energy, including the fact that it drops off with added distance from the Earth, $V = -G\frac{M_{earth} M_{satellite}}{r}$,

then the choice of zero potential is different. In this case we generally choose the zero of gravitational potential energy at infinity, since the gravitational force approaches zero at infinity, hence the reason why the above expression is negative. This is a sensible way to define the 'zero point' since the potential energy with respect to a point at infinity tells us the energy with which an object is fixed to the Earth. So, by decreasing its kinetic energy, its potential energy increases (becoming less negative) and can only do so when the radial distance, r decreases, thus forcing the satellite into a lower orbit. Once in this lower orbit, its speed $v = \sqrt{(G M_{earth} / r)}$
(see solution to question 8, part c).), therefore a reduced radius of orbit will result in an increase in the satellite's speed.

17. Gravity is still present. They still have a weight. The astronauts and space capsule are falling towards the earth however, resulting in the free fall "weightless" effect. When an object is in freefall, it experiences no Normal (other) force. "Weightlessness" is the absence of any other force to counterbalance the weight force. The seatbelt however provides an external resultant force which keeps the astronauts in position.

18. Kepler's law suggests that: $r^3 / T^2 = GM_{earth} / 4\pi^2$

$r^3 \times 4\pi^2 / GM_{earth} = T^2$ (assuming a polar orbit at an altitude of 550 km), then:

$T^2 = (5.50 \times 10^5 + 6.37 \times 10^6)^3 \times 4\pi^2/(6.67 \times 10^{-11} \times 5.97 \times 10^{24}) = 3.285 \times 10^7 \text{ s}^2$

$T = 5.73 \times 10^3$ s (1.59 hours)

19. a) Europa's orbital speed, $v = 2\pi r / T$

$v = 2 \pi\, 6.71 \times 10^8 / 3.07 \times 10^5$

$v = 1.37 \times 10^4 \text{ ms}^{-1}$ (13.7 kms^{-1})

b) Jupiter's mass, M_J, given by:

$r^3 / T^2 = G M_J / 4\pi^2$

$(6.71 \times 10^8)^3 / (3.07 \times 10^5)^2 = 6.67 \times 10^{-11} \times M_J / 4\pi^2$

$M_J = 1.90 \times 10^{27}$ kg

20. $r^3 / T^2 = G M_{earth} / 4\pi^2$

$(2.02 \times 10^7 + 6.37 \times 10^6)^3 / (12 \times 3600)^2 = 6.67 \times 10^{-11} \times M_{earth} / 4\pi^2$

$(2.657 \times 10^7)^3 / (43200)^2 = 6.67 \times 10^{-11} \times M_{earth} / 4\pi^2$

$M_{earth} = 5.95 \times 10^{24}$ kg

21. The satellite is geostationary. It has an orbital period of 24 hours (86400 s).

$r^3 / T^2 = G\,M_{earth} / 4\pi^2$, therefore: $r^3 / (24 \times 3600)^2 = 6.67 \times 10^{-11} \times 5.97 \times 10^{24} / 4\pi^2$

so, $r^3 = 6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times (86400)^2 / 4\pi^2$ gives $r^3 = 7.5295 \times 10^{22}$

therefore r = 4.223 x 10^7 m, suggesting that height above Earth, h = 3.59 x 10^7 m

22.

Radius of Moon - Earth orbit

$r_e{}^3 = G\,M_{earth} \times (27.3 \times 24 \times 3600)^2 / 4\pi^2$

Radius of Titan - Saturn orbit

$r_s{}^3 = G\,M_{saturn}\,T^2 / 4\pi^2$

$r_s{}^3 = G\,108M_{earth} \times (14 \times 24 \times 3600)^{22} / 4\pi^2$

Form ratio (fraction):

$$\frac{r_e{}^3}{r_s{}^3} = \frac{GM_{earth} \times (27.3 \times 24 \times 3600)^2 / 4\pi^2}{G\,108M_{earth} \times (14 \times 24 \times 3600)^2 / 4\pi^2}$$

$$\frac{r_e{}^3}{r_s{}^3} = \frac{(27.3)^2}{108 \times (14)^2}$$

$$\frac{r_e{}^3}{r_s{}^3} = 0.035208$$

take cube root of both sides

$r_e = 0.3278\, r_s \rightarrow$ $r_s = r_e/0.3277$ $\rightarrow$ $r_s = 3.05\, r_e$

$r_{\text{titan's orbit around Saturn}} = 3.05\, r_{\text{moon's orbit around Earth}}$ The ratio $r_{titan} : r_{moon}$ is 1:3.05

23. a) Without being provided with the mass of Mercury, only the **relative** force of attraction can be calculated:

Closest approach (perihelion)

$R_{peri} = 4.60 \times 10^{10}$ m

$F_{peri} = G\,M_{sun}\,M_{mercury} / R_{peri}{}^2$

Furthest approach (aphelion)

$R_{aphe} = 6.90 \times 10^{10}$ m

$F_{aphe} = G\,M_{sun}\,M_{mercury} / R_{aphe}{}^2$

Dividing both expressions cancels all the constant factors, leaving:

$$F_{peri} / F_{aphe} = R_{aphe}^2 / R_{peri}^2 = (6.9 \times 10^{10})^2 / (4.6 \times 10^{10})^2 = 2.25$$

so the attractive force at Mercury's perihelion due to the Sun is 2.25 times greater than the force of attraction at its aphelion

23. b) The respective velocities will also be different:

Closest approach (perihelion)

$R_{peri} = 4.60 \times 10^{10}$ m

$$v^2 = \frac{GM_{sun}}{R_{peri}}$$

$$v^2 = \frac{6.67 \times 10^{-11}\ 1.99 \times 10^{30}}{4.60 \times 10^{10}}$$

$v^2 = 2.886 \times 10^9\ m^2s^{-2}$

$v = 5.37 \times 10^4\ ms^{-1}$ ($53.7\ kms^{-1}$)

Furthest approach (aphelion)

$R_{aphe} = 6.90 \times 10^{10}$ m

$$v^{2\text{-}} = \frac{GM_{sun}}{R_{aphe}}$$

$$v^2 = \frac{6.67 \times 10^{-11}\ 1.99 \times 10^{30}}{6.90 \times 10^{10}}$$

$v^2 = 1.924 \times 10^9\ m^2s^{-2}$

$v = 4.39 \times 10^4\ ms^{-1}$ ($43.9\ kms^{-1}$)

c) Conservation of Energy. The energy possessed by Mercury is either in a gravitational potential form or in a translational kinetic energy form. None of its energy is lost to other objects (no energy transfer). If its total orbital energy remains constant then it will not decay (inward spiral) into the sun.